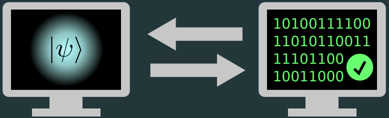


Classical verification of quantum computational advantage

Gregory D. Kahanamoku-Meyer

February 22, 2022

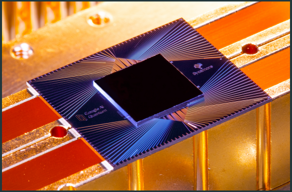
arXiv:2104.00687 (theory)
arXiv:2112.05156 (expt.)

Theory collaborators:

Norman Yao (UCB → Harvard)
Umesh Vazirani (UCB)
Soonwon Choi (UCB → MIT)

Quantum computational advantage

Recent experimental demonstrations:



Random circuit sampling
[Arute et al., Nature '19]

Gaussian boson sampling
[Zhong et al., Science '20]

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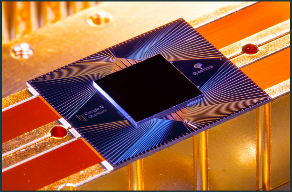

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Largest experiments $\rightarrow$ impossible to classically simulate

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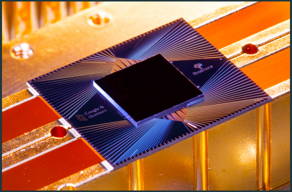

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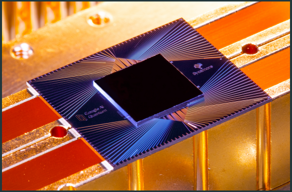

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Quantum is the only reasonable explanation for observed behavior

“Black-box” quantum computational advantage

Stronger: rule out all classical hypotheses, even pathological!

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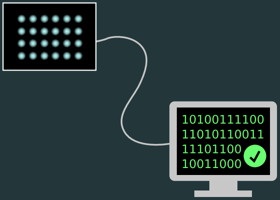

Local: powerfully refute the extended Church-Turing thesis

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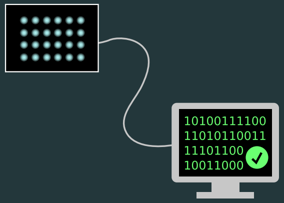

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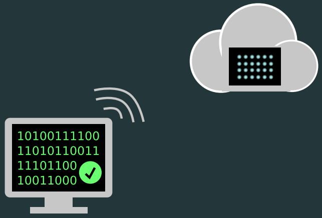

Remote: validate an untrusted quantum cloud service

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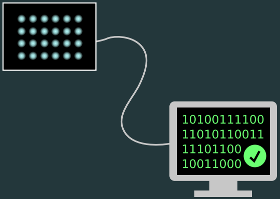

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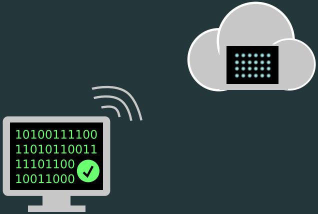

Remote: validate an untrusted quantum cloud service

Proof not specific to quantum mechanics: disprove null hypothesis that output was generated classically.

NISQ verifiable quantum advantage

Trivial solution: Shor's algorithm

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Trivial solution: Shor's algorithm... but we want to do near-term!

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NISQ: Noisy Intermediate-Scale Quantum devices

Sampling problems
e.g. random circuits, Boson sampling, ...
✓ NISQ feasible
✗ Efficiently verifiable

Number theory problems
e.g. factoring, discrete logarithm, ...
✗ NISQ feasible
✓ Efficiently verifiable

add structure

make less costly

???
✓ NISQ feasible
✓ Efficiently verifiable

Making number theoretic problems less costly

Fully solving a problem like factoring is “overkill”

Making number theoretic problems less costly

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Can we demonstrate quantum *capability* without needing to solve such a hard problem?

Zero-knowledge proofs: differentiating colors

Challenge: You have a friend who is red/green colorblind. How do you convince them that a red and a green ball that appear identical are different?

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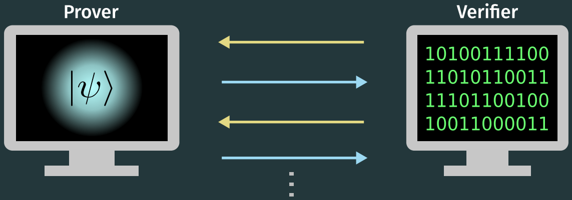
Color blind friend $\Leftrightarrow$ Classical verifier
Seeing color $\Leftrightarrow$ Quantum capability

Interactive proofs of quantumness

Multiple rounds of interaction between the prover and verifier



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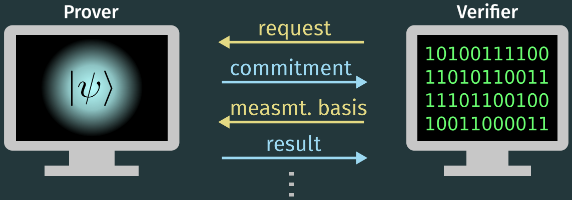

Round 1: Prover commits to a specific quantum state

Round 2: Verifier asks for measurement in specific basis

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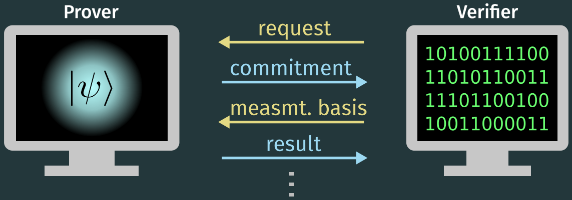

Round 1: Prover commits to a specific quantum state

Round 2: Verifier asks for measurement in specific basis

By randomizing choice of basis and repeating interaction, can ensure prover would respond correctly in *any* basis

Brakerski, Christiano, Mahadev, Vidick, Vazirani '18 (arXiv:1804.00640).

Can be extended to verify arbitrary quantum computations! (arXiv:1804.01082)

State commitment (round 1): trapdoor claw-free functions

How does the prover commit to a state?

Consider a **2-to-1** function f:
for all y in range of f, there exist (x_0, x_1) such that $y = f(x_0) = f(x_1)$.

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Evaluate f on uniform superposition $\sum_x \|x\rangle \|f(x)\rangle$	$\xleftarrow{\quad f \quad}$	Pick 2-to-1 function f
Measure 2nd register as y	$\xrightarrow{\quad y \quad}$	Store y as commitment

Prover has committed to the state $(|x_0\rangle + |x_1\rangle)\,|y\rangle$

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Prover has committed to $(|x_0\rangle + |x_1\rangle)|y\rangle$ with $y = f(x_0) = f(x_1)$

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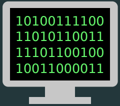
The only path to a valid state without trapdoor is by superposition + wavefunction collapse—inherently quantum!

Brakerski, Christiano, Mahadev, Vazirani, Vidick '18

Prover

Verifier



Pick trapdoor claw-free function f

$\xleftarrow{\ f\ }$

Evaluate f on uniform superposition: $\sum_x |x\rangle |f(x)\rangle$

Measure 2nd register as y

$\xrightarrow{\ y\ }$

Compute x_0, x_1 from y using trapdoor

arXiv:1804.00640. Can be extended to verify arbitrary quantum computations! arXiv:1804.01082

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Learning-with-Errors TCF has adaptive hardcore bit

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Trapdoor claw-free functions

TCF	Trapdoor	Claw-free	Adaptive hard-core bit
LWE [1]	✓	✓	✓
Ring-LWE [2]	✓	✓	✗
$x^2 \bmod N$ [3]	✓	✓	✗
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Interactive measurement: computational Bell test

Replace X basis measurement with two-step process:
"condense" x_0, x_1 into a single qubit, and then do a "Bell test."

$\vdots$ $\quad\vdots\quad$ $\vdots$

$|x_0\rangle |x_0 \cdot r\rangle + |x_1\rangle |x_1 \cdot r\rangle$ $\quad \xleftarrow{\quad r \quad} \quad$ Pick random bitstring r

Measure all but ancilla in X basis $\quad \xrightarrow{\quad d \quad}$

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Now single-qubit state: $|0\rangle$ or $|1\rangle$ if $x_0 \cdot r = x_1 \cdot r$, otherwise $|+\rangle$ or $|-\rangle$.
Polarization hidden via:

Cryptographic secret (here) $\Leftrightarrow$ Non-communication (Bell test)

GDKM, Choi, Vazirani, Yao '21 (arXiv:2104.00687)

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Now can use any trapdoor claw-free function!

GDKM, Choi, Vazirani, Yao '21 (arXiv:2104.00687)

Computational Bell test: classical bound

Run protocol many times, collect statistics.

p_Z: Success rate for Z basis measurement.

p_{CHSH}: Success rate when performing CHSH-type measurement.

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Classical bound: $p_Z + 4p_{\text{CHSH}} - 4 < \text{negl}(n)$

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Note: Let $p_Z = 1$. Then for p_{CHSH}:

Classical bound 75%, ideal quantum $\sim$ 85%. Same as regular CHSH!

GDKM, Choi, Vazirani, Yao '21 (arXiv:2104.00687)

Moving towards full efficiently-verifiable quantum adv. on NISQ

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Interaction

- Intermediate measurement: need to measure subsystem while maintaining coherence on other qubits

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- Measurement-based uncomputation scheme
- … hopefully can continue making theory improvements!

Intermediate measurements in the lab

Trapped Ion Quantum Information lab at U. Maryland ($\to$ Duke)

First demonstration of protocols, in trapped ions! (arXiv:2112.05156)

Dr. Daiwei Zhu

Prof. Crystal Noel

Prof. Christopher Monroe

and others!

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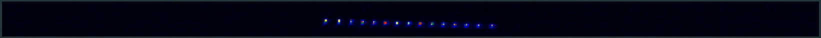
First demonstration of protocols, in trapped ions! (arXiv:2112.05156)

Partial measurement:



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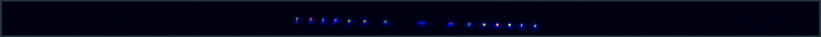

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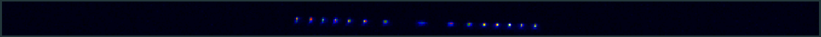

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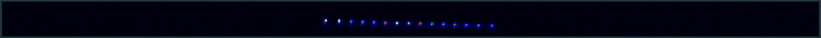
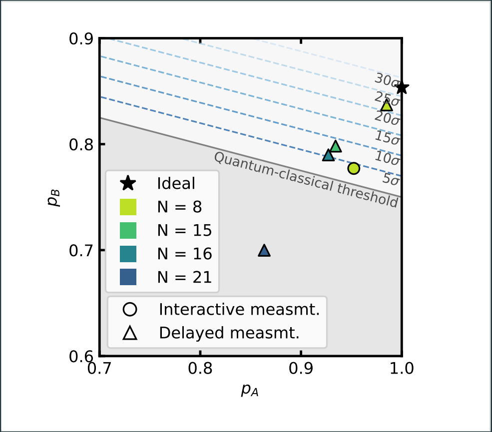


Interactive proofs on a few qubits



GDKM, D. Zhu, et al. (arXiv:2112.05156)

Technique: postselection

How to deal with high fidelity requirement? Naively need $\sim 83\%$ overall circuit fidelity to pass.

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When we generate $\sum_x |x\rangle\,|f(x)\rangle$, **add redundancy to $f(x)$, for bit flip error detection!**

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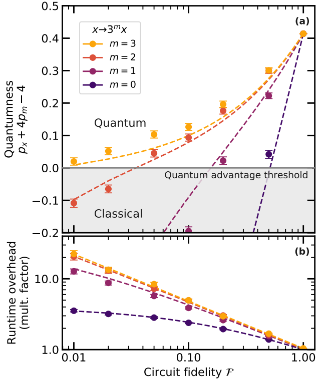

Numerical results for $x^2 \bmod N$ with $\log N = 512$ bits.

Here: make transformation $x^2 \bmod N \Rightarrow (kx)^2 \bmod k^2 N$

Improving circuit sizes

Most demanding step in all these protocols: evaluating TCF

$$\mathcal{U}_f \left|x\right\rangle \left|0^{\otimes n}\right\rangle = \left|x\right\rangle \left|f(x)\right\rangle$$

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Getting rid of adaptive hardcore bit helps!

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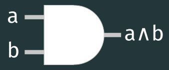
Protocol allows us to make circuits irreversible!

Technique: taking out the garbage

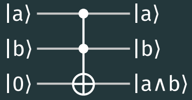
Goal: $\mathcal{U}_f \left|x\right\rangle \left|0^{\otimes n}\right\rangle = \left|x\right\rangle \left|f(x)\right\rangle$

When converting classical circuits to quantum:

Garbage bits: extra entangled outputs due to unitarity



Classical AND



Quantum AND (Toffoli)

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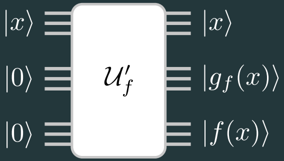

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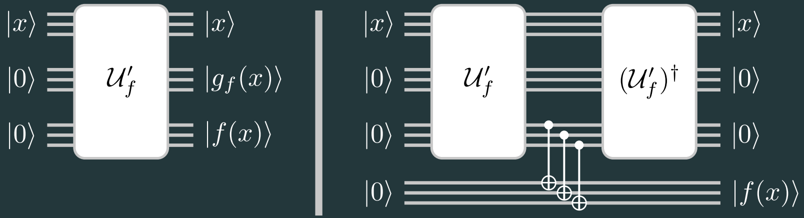

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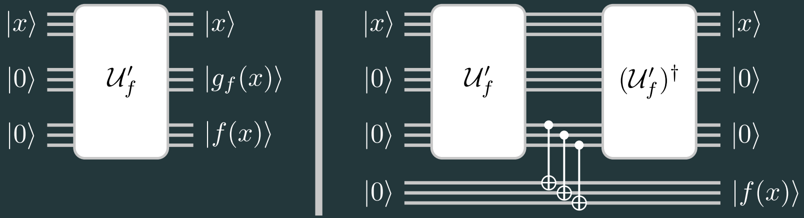

Lots of time and space overhead!

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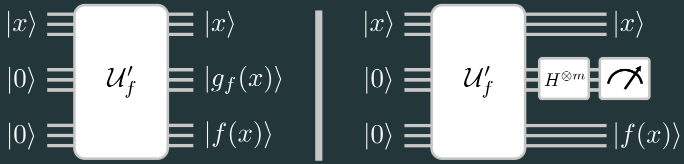

Can we "measure them away" instead?

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Measure garbage bits $g_f(x)$ in X basis, get some string h. End up with state:

$$\sum_x (-1)^{h \cdot g_f(x)} |x\rangle |f(x)\rangle$$

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1024-bit $x^2 \mod N$ in depth 10^5 (and can be improved?)

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Advantages:

- Everything is diagonal (it's just a phase)!
- Modulo is automatic in the phase
- Simple decomposition into few-qubit gates

Implementation

New goal: $\tilde{\mathcal{U}} \left|x\right\rangle \left|z\right\rangle = \exp\left(2\pi i \frac{x^2}{N} z\right) \left|x\right\rangle \left|z\right\rangle$

Decompose using “grade school” integer multiplication:

$$a \cdot b = \sum_{i,j} 2^{i+j} a_i b_j$$

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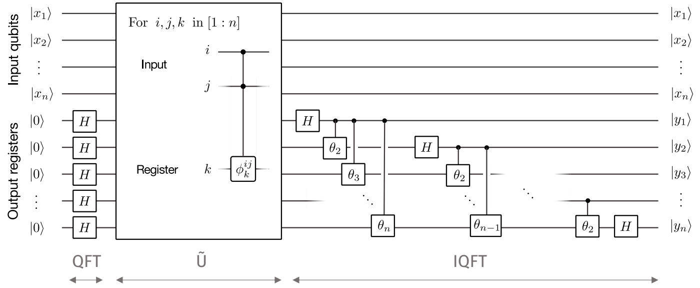
- These are CCPhase gates (of arb. phase)!

Leveraging the Rydberg blockade



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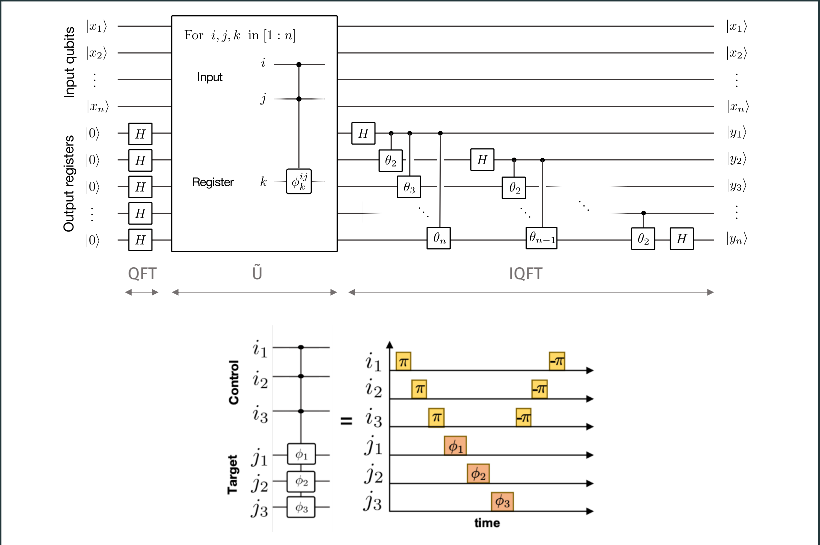

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Bottleneck: Evaluating TCF on quantum superposition

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Way outside the box?

Backup!

NISQ verifiable quantum advantage

NISQ: Noisy Intermediate-Scale Quantum devices

Sampling problems

e.g. random circuits, Boson sampling, ...

- ✓ NISQ feasible
- ✗ Efficiently verifiable

Number theory problems

e.g. factoring, discrete logarithm, ...

- ✗ NISQ feasible
- ✓ Efficiently verifiable

add structure

make less costly

???

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Adding structure opens opportunities for classical cheating

Decisional Diffie-Hellman (DDH)

Problem (not TCF): Consider a group $\mathbb{G}$ of order N, with generator g. Given the tuple (g, g^a, g^b, g^c), determine if $c = ab$.

Elliptic curve crypto.: $\log N \sim 160$ bits is as hard as 1024 bit factoring!!

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Trapdoor [Peikert, Waters '08; Freeman et al. '10]: linear algebra in the exponent

Claw-free [GDKM et al. '21 (arXiv:2104.00687)]: collisions in linear algebra in the exponent!

Full protocol

Prover (quantum)		Verifier (classical)
		
Round 1	$\xleftarrow{f_i}$	1. Sample $(f_i, t) \leftarrow \mathsf{Gen}(1^n)$
2. Generate state $\sum_{x=0}^{N/2} \|x\rangle_{\mathsf{x}} \|f_i(x)\rangle_{\mathsf{y}}$		
3. Measure y register, yielding bitstring y State is now $(\|x_0\rangle + \|x_1\rangle)_{\mathsf{x}} \|y\rangle_{\mathsf{y}}$; y register can be discarded	$\xrightarrow{y}$	4. Using trapdoor t compute x_0 and x_1
	$\xleftarrow{\text{choice}}$	5. Randomly choose to request a preimage or continue
If preimage requested:		
6a. Projectively measure x register, yielding x	$\xrightarrow{x}$	7a. If $x \in \{x_0, x_1\}$ return Accept
Otherwise, continue:		
Round 2	$\xleftarrow{r}$	6b. Choose random bitstring r
7b. Add one ancilla b; use CNOTs to compute $\|r \cdot x_0\rangle_{\mathsf{b}} \|x_0\rangle_{\mathsf{x}} + \|r \cdot x_1\rangle_{\mathsf{b}} \|x_1\rangle_{\mathsf{x}}$ where $r \cdot x$ is bitwise inner product		
8b. Measure x register in Hadamard basis, yielding a string d. Discard x, state is now $\|\psi\rangle_{\mathsf{b}} \in \{\|0\rangle, \|1\rangle, \|+\rangle, \|-\rangle\}$	$\xrightarrow{d}$	9b. Using r, x_0, x_1, d, determine $\|\psi\rangle_{\mathsf{b}}$
Round 3	$\xleftarrow{m}$	10b. Choose random $m \in \{\frac{\pi}{4}, -\frac{\pi}{4}\}$
11b. Measure ancilla b in the rotated basis $\left\{\begin{matrix}\cos\left(\frac{m}{2}\right)\|0\rangle + \sin\left(\frac{m}{2}\right)\|1\rangle \\ \cos\left(\frac{m}{2}\right)\|1\rangle - \sin\left(\frac{m}{2}\right)\|0\rangle\end{matrix}\right\}$, yielding a bit b	$\xrightarrow{b}$	11b. If b was likely given $\|\psi\rangle_{\mathsf{b}}$ return Accept