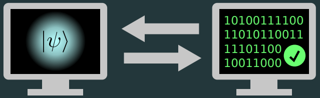
Cryptographic protocols for classically-verifiable quantum advantage and more



Gregory D. Kahanamoku-Meyer

March 1, 2023

Berkeley
UNIVERSITY OF CALIFORNIA

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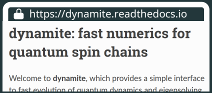

https://dynamite.readthedocs.io

dynamite: fast numerics for quantum spin chains

Welcome to **dynamite**, which provides a simple interface to fast evolution of quantum dynamics and eigensolving

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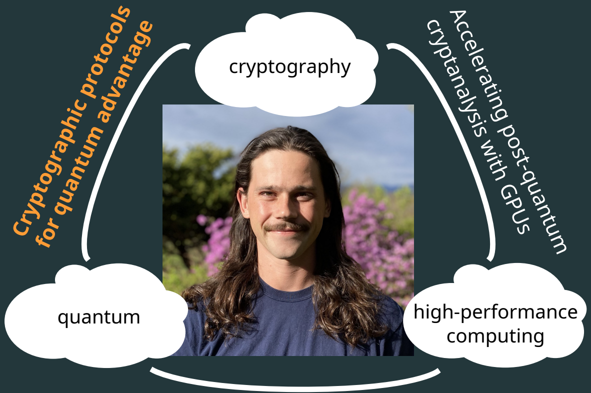

cryptography

quantum

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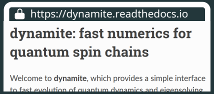
Cryptographic protocols for quantum advantage

Accelerating post-quantum cryptanalysis with GPUs



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Quantum computational advantage

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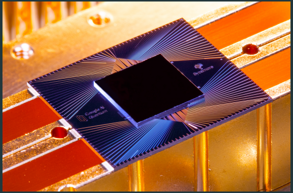

Random circuit sampling
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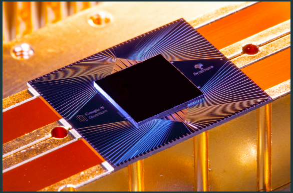

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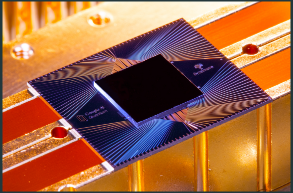

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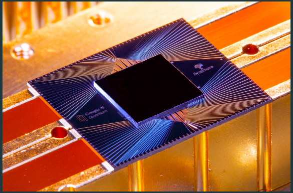

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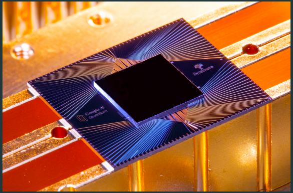

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Biggest experiments impossible to classically simulate—how do we verify the output?

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Quantum is the only reasonable explanation for observed behavior,
under some assumptions about the inner workings of the device

“Black-box” quantum computational advantage

Stronger: rule out all classical hypotheses, even pathological!

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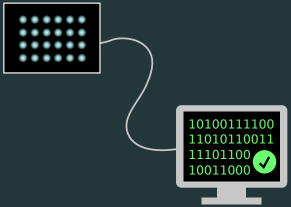

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"Qubits prove their power to humanity"

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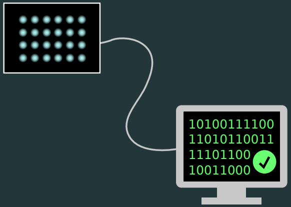

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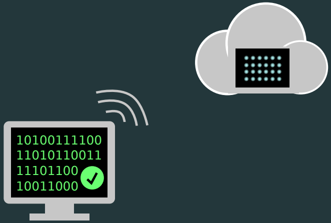

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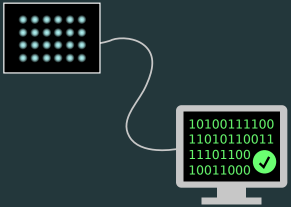

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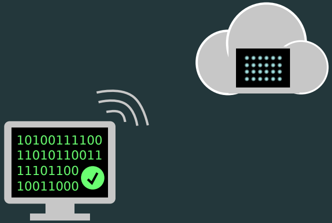

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Reframing: disprove null hypothesis that output was generated classically.

Noisy intermediate scale verifiable quantum advantage

Trivial solution: Shor's algorithm

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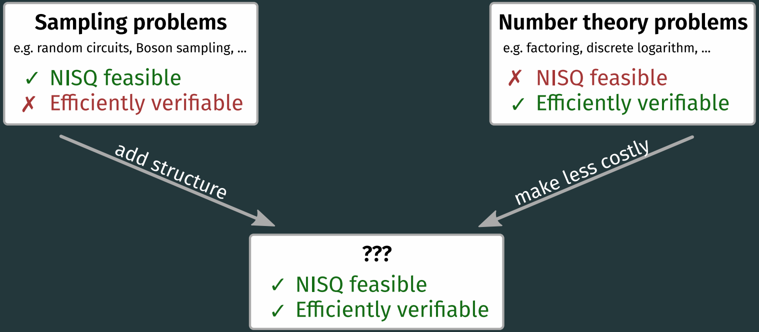
NISQ: Noisy Intermediate-Scale Quantum devices



Sampling problems

e.g. random circuits, Boson sampling, ...

✓ NISQ feasible
✗ Efficiently verifiable

Number theory problems

e.g. factoring, discrete logarithm, ...

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add structure

make less costly

???

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Example: sampling “IQP” circuits (products of Pauli X’s)

$$H = X_0 X_1 X_3 + X_1 X_2 X_4 X_5 + \cdots \tag{1}$$

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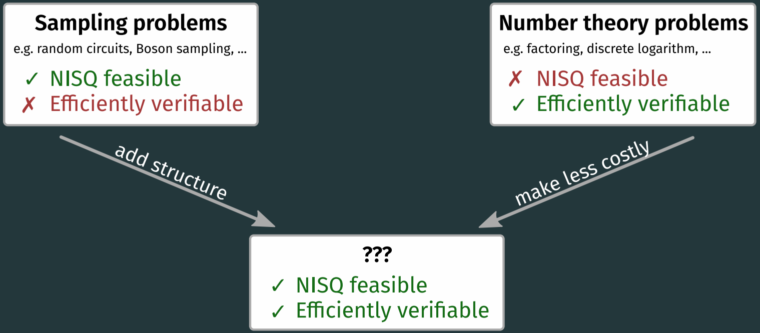
Adding structure opens opportunities for classical cheating

Noisy intermediate scale verifiable quantum advantage



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Making number theoretic problems less costly

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Can we demonstrate quantum *capability* without needing to solve such a hard problem?

Zero-knowledge proofs: differentiating colors

You are red/green colorblind, your friend is not.

How can they use a red ball and green ball to convince you that they see color?

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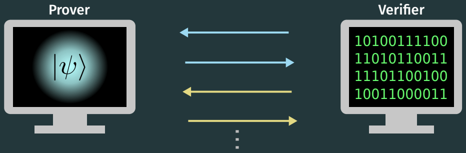
Goal: find protocol **as verifiable and classically hard as factoring**—
but **less expensive than actually finding factors (via Shor)**

Interactive proofs of quantumness

Multiple rounds of interaction between the prover and verifier



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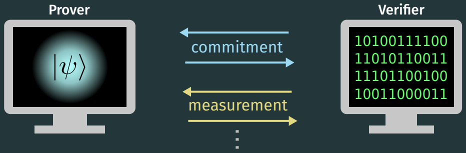

Round 1: Prover commits to holding a specific quantum state

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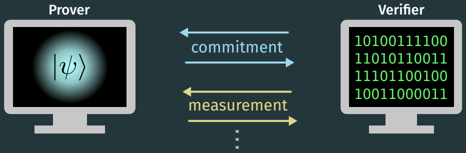

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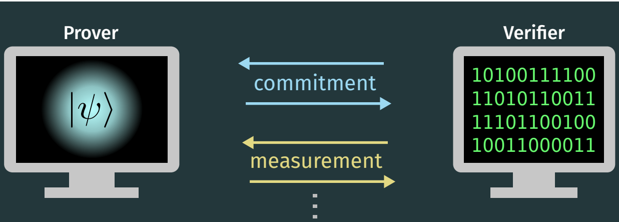
By randomizing choice of basis and repeating interaction,
can ensure prover would respond correctly in *any* basis

Brakerski, Christiano, Mahadev, Vidick, Vazirani ’18 (arXiv:1804.00640).

Can be extended to verify arbitrary quantum computations! (arXiv:1804.01082)

Hardness proof: rewinding



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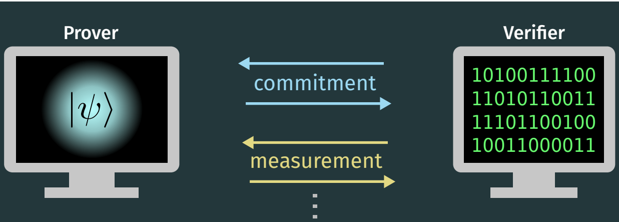

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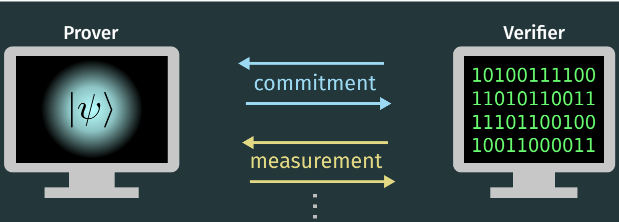

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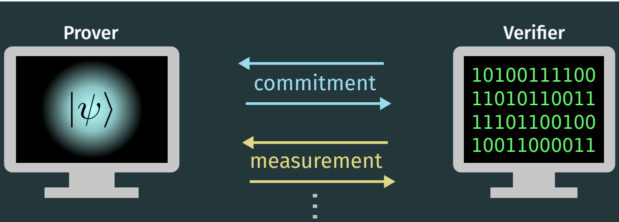

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“Rewinding” proof of hardness doesn’t go through for quantum prover—can even use functions that are quantum claw-free!

State commitment (round 1): trapdoor claw-free functions

How does the prover commit to a state?

Consider a **2-to-1** function f:
for all y in range of f, there exist (x_0, x_1) such that $y = f(x_0) = f(x_1)$.

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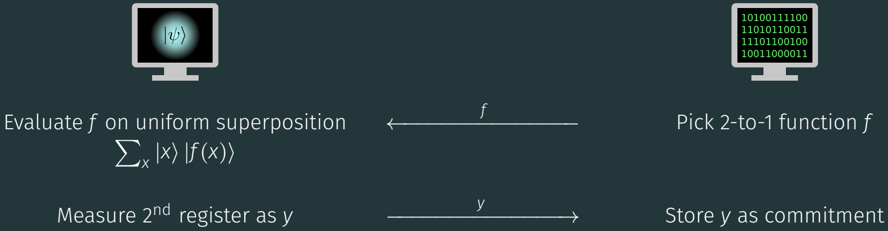

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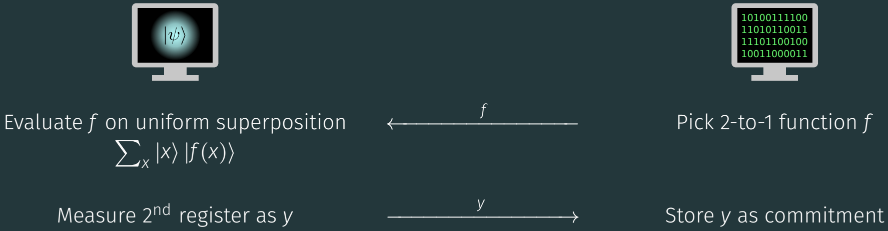

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Generating a valid state without trapdoor uses superposition + wavefunction collapse—inherently quantum!

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$$f(x) = x^2 \bmod N, \text{ where } N = pq$$

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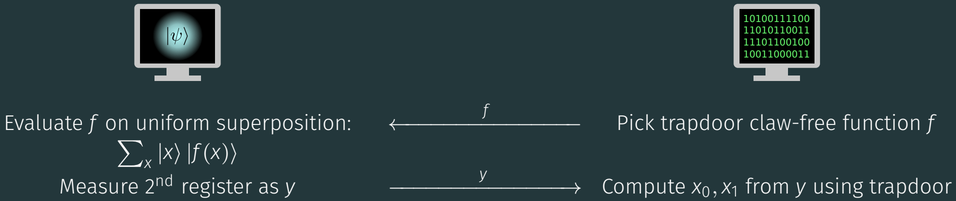
Example: $4^2 \equiv 11^2 \equiv 16 \pmod{35}$; and $11 - 4 = 7$

Brakerski, Christiano, Mahadev, Vazirani, Vidick ’18



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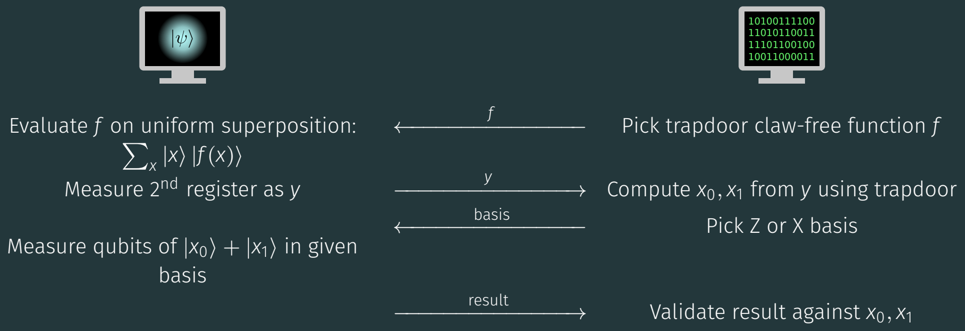

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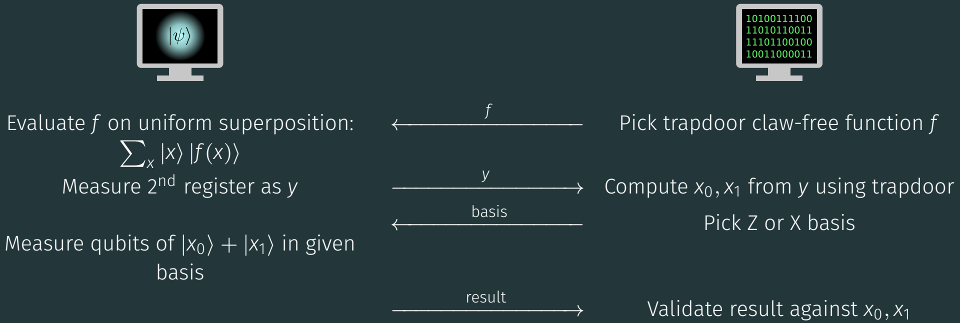

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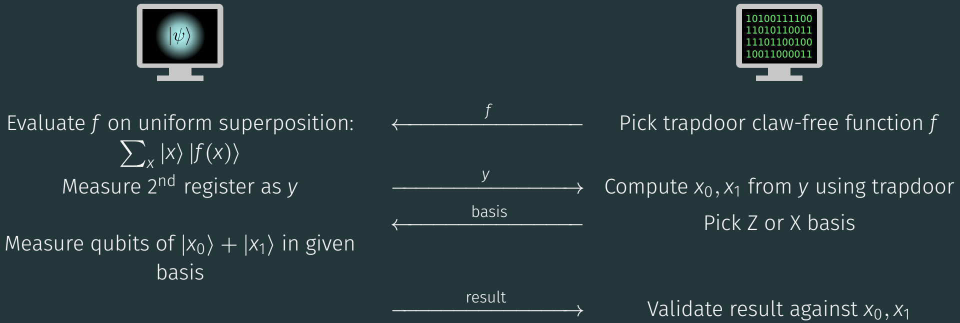
Brakerski, Christiano, Mahadev, Vazirani, Vidick ’18



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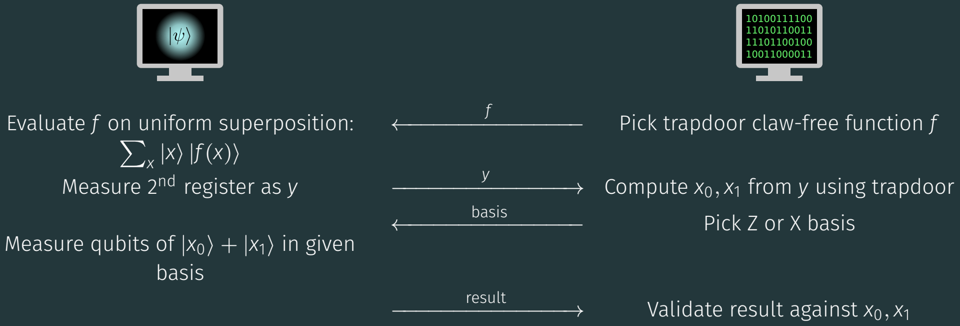

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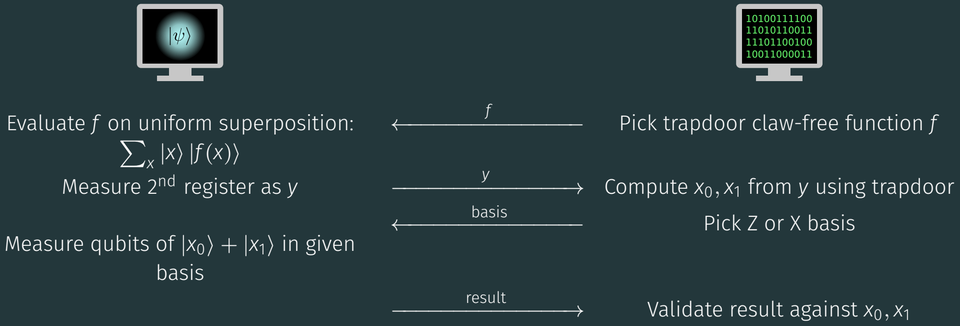

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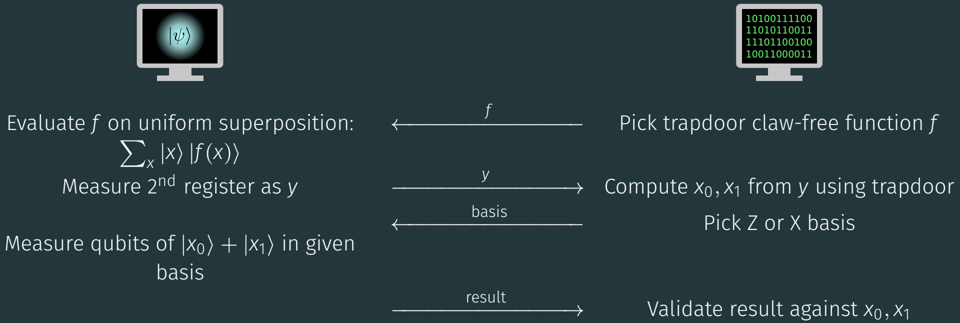

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Protocol requires strong claw-free property:

For any x_0, hard to find even **a single bit** about x_1.

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Trapdoor claw-free functions

Function family	Trapdoor	Claw-free	Strong claw-free
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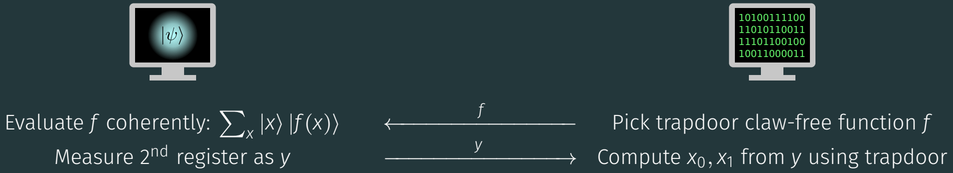

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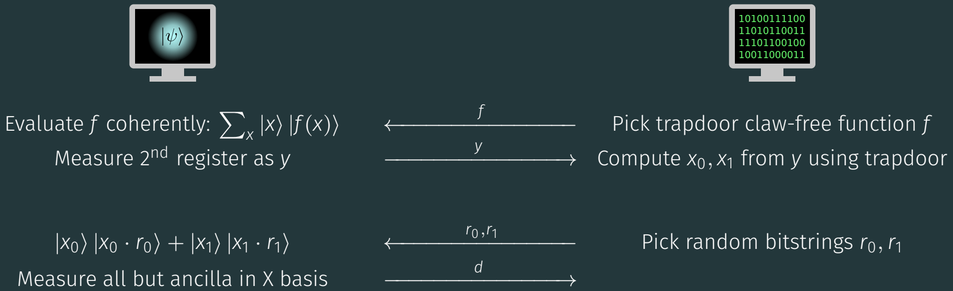

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Cryptographic secret (here) $\Leftrightarrow$ Non-communication (Bell test)

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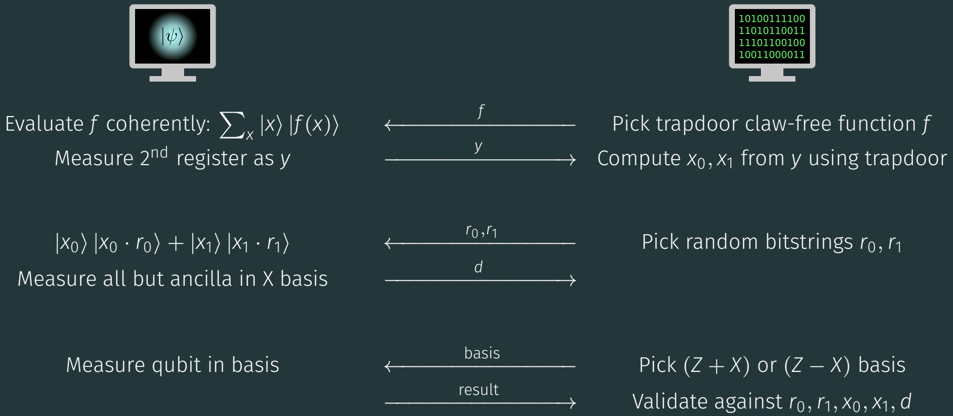

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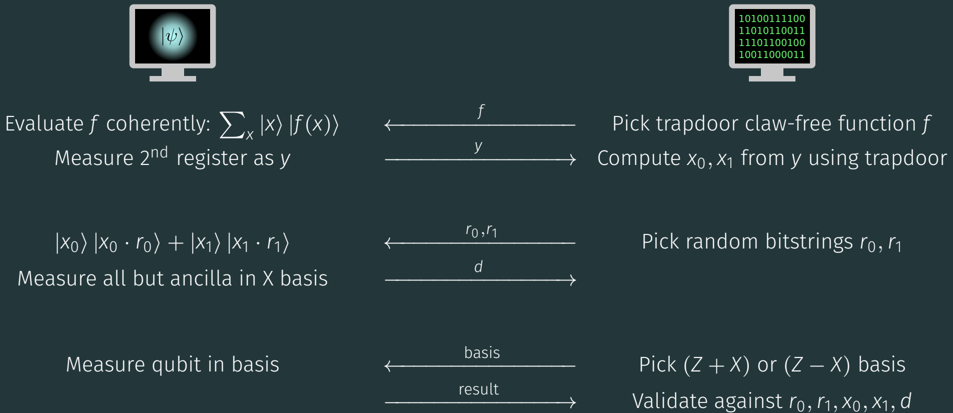

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This protocol can use any trapdoor claw-free function!

Computational Bell test: classical bound

Let p be the probability that the prover succeeds in a single iteration of the protocol.

Under assumption of claw-free function:

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Next up: tricks for the near term

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When we generate $\sum_x |x\rangle\,|f(x)\rangle$, **add redundancy to $f(x)$, for bit flip error detection!**

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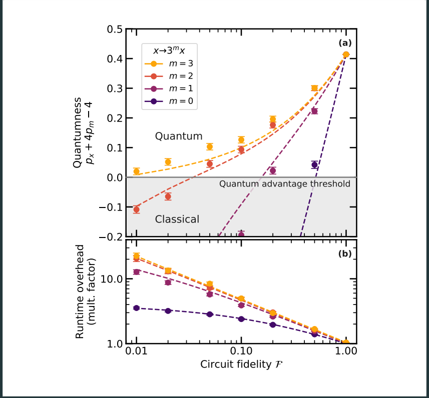

Numerical results for $x^2 \bmod N$ with $\log N = 512$ bits.

Here: make transformation $x^2 \bmod N \Rightarrow (kx)^2 \bmod k^2 N$

Improving circuit sizes

Most demanding step in all these protocols: evaluating TCF

$$\mathcal{U}_f \left|x\right\rangle \left|0^{\otimes n}\right\rangle = \left|x\right\rangle \left|f(x)\right\rangle$$

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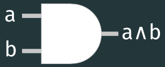
Protocol allows us to make circuits irreversible!

Technique: taking out the garbage

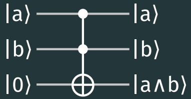
Goal: $\mathcal{U}_f \left|x\right\rangle \left|0^{\otimes n}\right\rangle = \left|x\right\rangle \left|f(x)\right\rangle$

When converting classical circuits to quantum:

Garbage bits: extra entangled outputs due to unitarity



Classical AND



Quantum AND (Toffoli)

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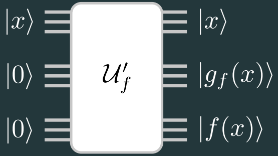

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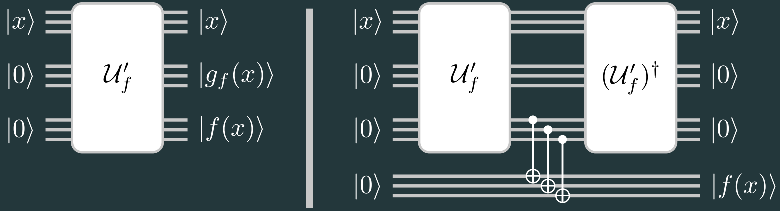

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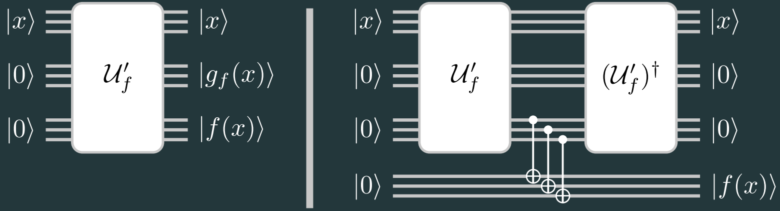

Lots of time and space overhead!

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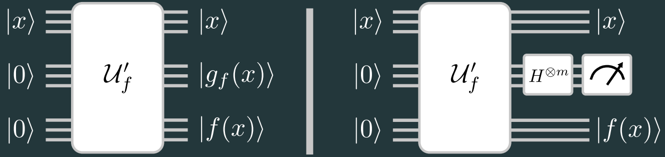

Can we “measure them away” instead?

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Measure garbage bits $g_f(x)$ in X basis, get some string h. End up with state:

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Can directly convert classical circuits to quantum!

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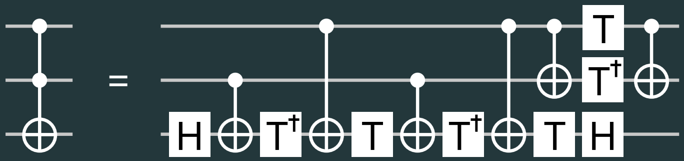

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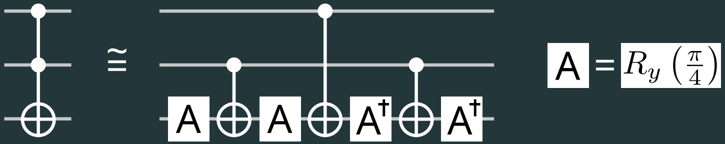

Summary + challenge

Rules of the game:

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I bet we can do better!

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Without post-quantum cryptography: not really

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New results: Brakersi, Gheorghiu, GDKM, Porat, Vidick ’23 (will be on arXiv imminently!)

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Takeaway: **protocol can “certify a qubit”**

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- (likely) **Classical, cryptographic verification of remote quantum computation!** (cf. Natarajan + Zhang, also about to post!)

Looking forward

Interactive cryptographic protocols:

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- **Longer term:** cryptographic applications!

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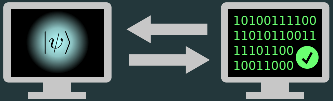
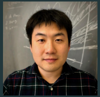
Questions?



"Classically verifiable quantum advantage from a computational Bell test"
[arXiv:2104.00687]

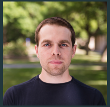
Norman Yao


Soonwon Choi

Umesh Vazirani

"Simple tests of quantumness also certify qubits" [on arXiv soon!]

Zvika Brakerski


Andru Gheorghiu

Eitan Porat

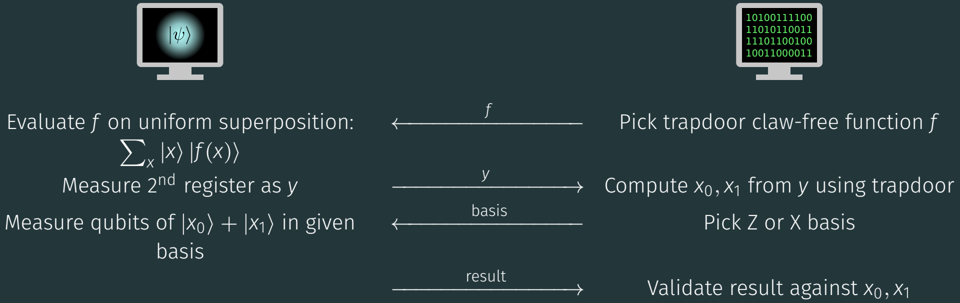
Thomas Vidick

Gregory D. Kahanamoku-Meyer

https://gregdmeyer.github.io/

Backup!

Brakerski, Christiano, Mahadev, Vazirani, Vidick ’18



Evaluate f on uniform superposition: $\sum_x |x\rangle |f(x)\rangle$ $\xleftarrow{f}$ Pick trapdoor claw-free function f

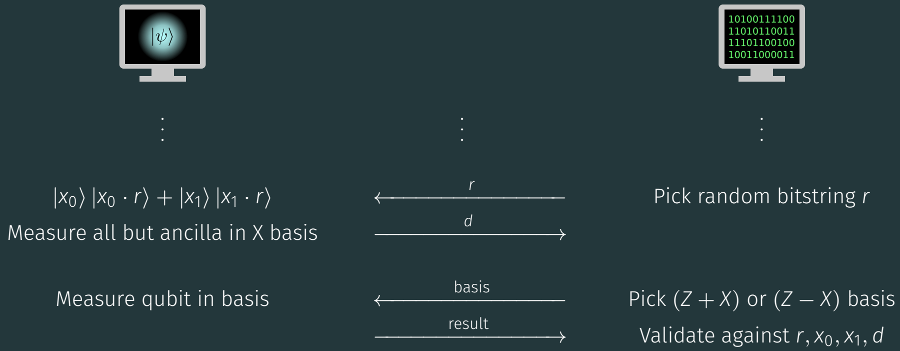
Measure 2nd register as y $\xrightarrow{y}$ Compute x_0, x_1 from y using trapdoor

Measure qubits of $|x_0\rangle + |x_1\rangle$ in given basis $\xleftarrow{\text{basis}}$ Pick Z or X basis

$\xrightarrow{\text{result}}$ Validate result against x_0, x_1

arXiv:1804.00640

Interactive measurement: computational Bell test



$|x_0\rangle\,|x_0 \cdot r\rangle + |x_1\rangle\,|x_1 \cdot r\rangle$ $\xleftarrow{r}$ Pick random bitstring r

Measure all but ancilla in X basis $\xrightarrow{d}$

Measure qubit in basis $\xleftarrow{\text{basis}}$ Pick $(Z+X)$ or $(Z-X)$ basis

$\xrightarrow{\text{result}}$ Validate against r, x_0, x_1, d

In this case, 1-qubit state: $|0\rangle$ or $|1\rangle$ if $x_0 \cdot r = x_1 \cdot r$, otherwise $|+\rangle$ or $|-\rangle$.

GDKM, Choi, Vazirani, Yao '21 (arXiv:2104.00687)

Computational Bell test: classical bound

Run protocol many times, collect statistics.

p_Z: Success rate for Z basis measurement.

p_{Bell}: Success rate when performing Bell-type measurement.

Under assumption of claw-free function:

Classical bound: $p_Z + 4p_{\text{Bell}} \lesssim 4$

Ideal quantum: $p_Z = 1, p_{\text{Bell}} = \cos^2(\pi/8)$

$$p_Z + 4p_{\text{Bell}} = 3 + \sqrt{2} \approx \mathbf{4.414}$$

Note: Let $p_Z = 1$. Then for p_{Bell}:

Classical bound 75%, ideal quantum $\sim$ 85%.

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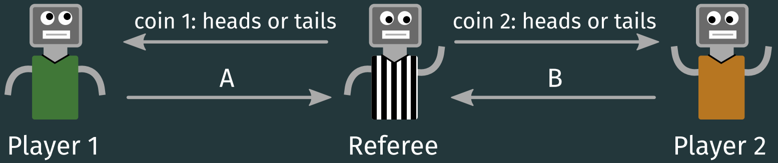
The CHSH game (Bell test)

Cooperative two-player game; players can't communicate (non-local).



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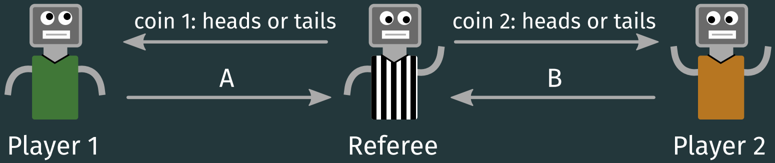

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Classical optimal strategy: return equal values, hope you didn't both get heads. 75% success rate.

Can we do better with entanglement?

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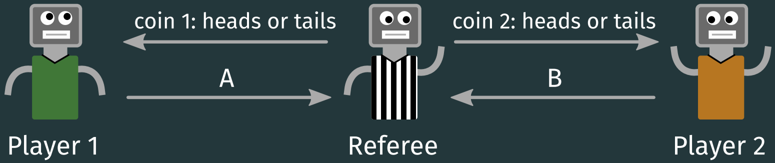

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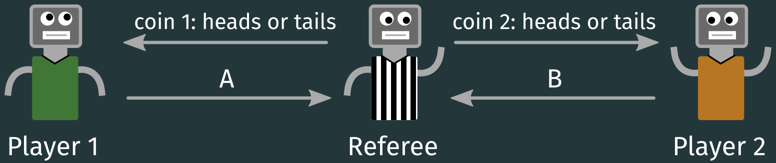

coin 1: heads or tails

A

coin 2: heads or tails

B

Player 1

Referee

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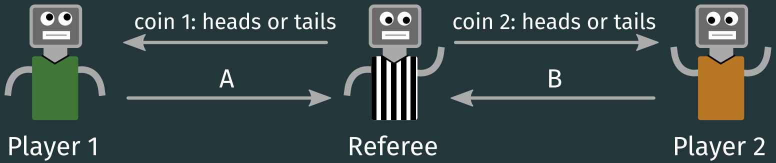

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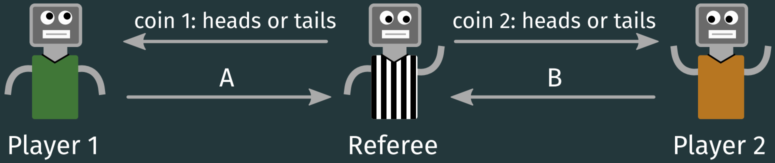

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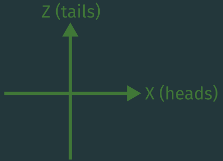

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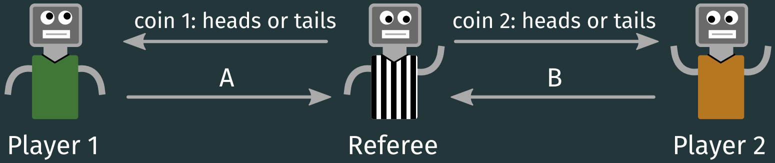

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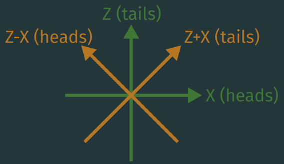

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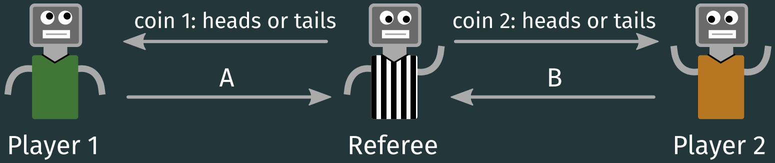

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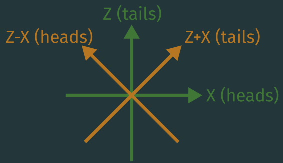

Quantum: $\cos^2(\pi/8) \approx 85\%$

Classical: 75%

Intermediate measurements in the lab

TIQI

Trapped Ion Quantum Information lab at U. Maryland (→ Duke)

First proof-of-concept demonstration of these protocols, in trapped ions! (arXiv:2112.05156)

Dr. Daiwei Zhu

Prof. Crystal Noel

Prof. Christopher Monroe

and others!

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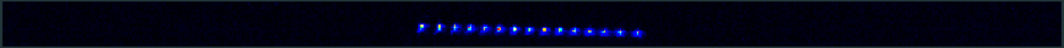

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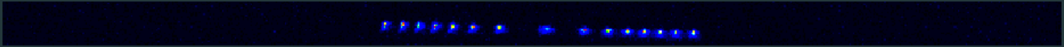

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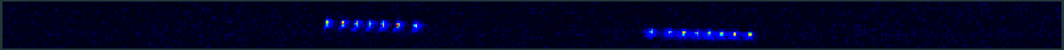

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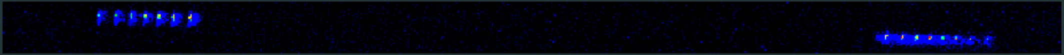

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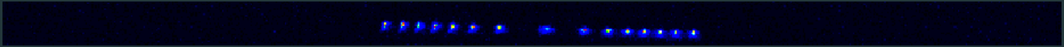

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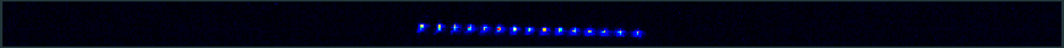
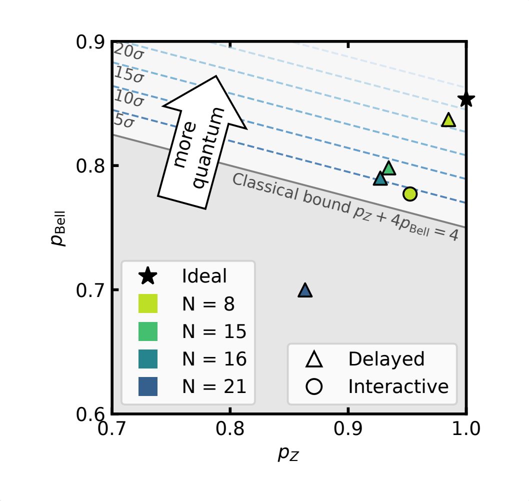


Interactive proofs on a few qubits

Experimental results for $f(x) = x^2 \mod N$

Up and **right** is stronger evidence of quantumness

GDKM, D. Zhu, et al. (arXiv:2112.05156)



Quantum circuits for $x^2 \bmod N$

Goal: $\mathcal{U}\,|x\rangle\,|0\rangle = |x\rangle\,\left|x^2 \bmod N\right\rangle$

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Decompose this as

$$\mathcal{U} = (\mathbb{I} \otimes \mathrm{IQFT}_N) \cdot \tilde{\mathcal{U}} \cdot (\mathbb{I} \otimes \mathrm{QFT}_N)$$

with

$$\tilde{\mathcal{U}}|x\rangle|z\rangle = \exp\left(2\pi i \frac{x^2}{N} z\right)|x\rangle|z\rangle$$

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Advantages:

- Everything is diagonal (it's just a phase)!
- Modulo is automatic in the phase
- Simple decomposition into few-qubit gates

Implementation

New goal: $\tilde{\mathcal{U}} \left|x\right\rangle \left|z\right\rangle = \exp\left(2\pi i \frac{x^2}{N} z\right) \left|x\right\rangle \left|z\right\rangle$

Decompose using “grade school” integer multiplication:

$$a \cdot b = \sum_{i,j} 2^{i+j} a_i b_j$$

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Implementation

New goal: $\tilde{\mathcal{U}} \left|x\right\rangle \left|z\right\rangle = \exp\left(2\pi i \frac{x^2}{N} z\right) \left|x\right\rangle \left|z\right\rangle$

Decompose using “grade school” integer multiplication:

$$a \cdot b = \sum_{i,j} 2^{i+j} a_i b_j$$

$$x^2 z = \sum_{i,j,k} 2^{i+j+k} x_i x_j z_k$$

$$\exp\left(2\pi i \frac{x^2}{N} z\right) = \prod_{i,j,k} \exp\left(2\pi i \frac{2^{i+j+k}}{N} x_i x_j z_k\right)$$

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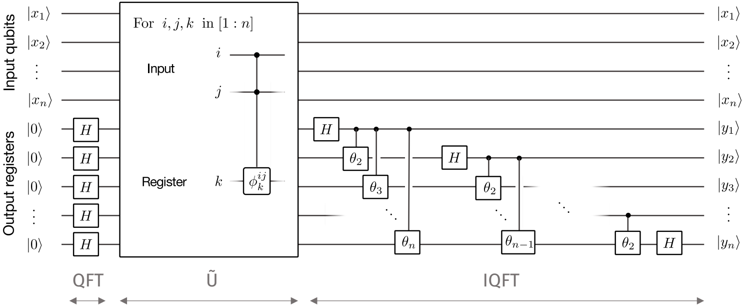
- Binary multiplication is AND
- “Apply phase whenever $x_i = x_j = z_k = 1$”
- These are CCPhase gates (of arb. phase)!

Leveraging the Rydberg blockade



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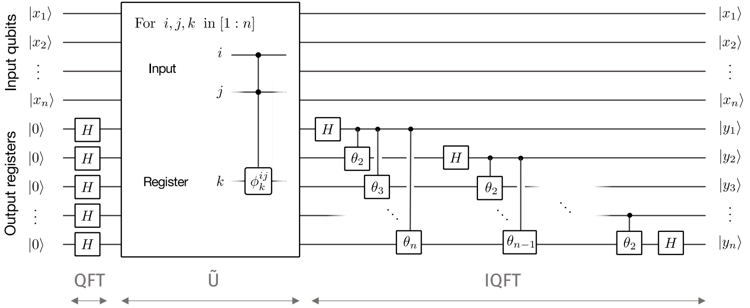

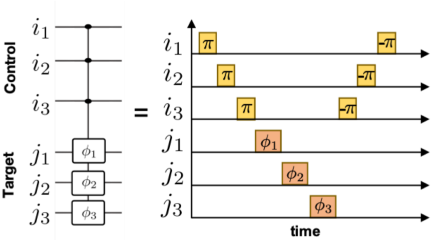

Decisional Diffie-Hellman (DDH)

Problem (not TCF): Consider a group $\mathbb{G}$ of order N, with generator g. Given the tuple (g, g^a, g^b, g^c), determine if $c = ab$.

Elliptic curve crypto.: $\log N \sim 160$ bits is as hard as 1024 bit factoring!!

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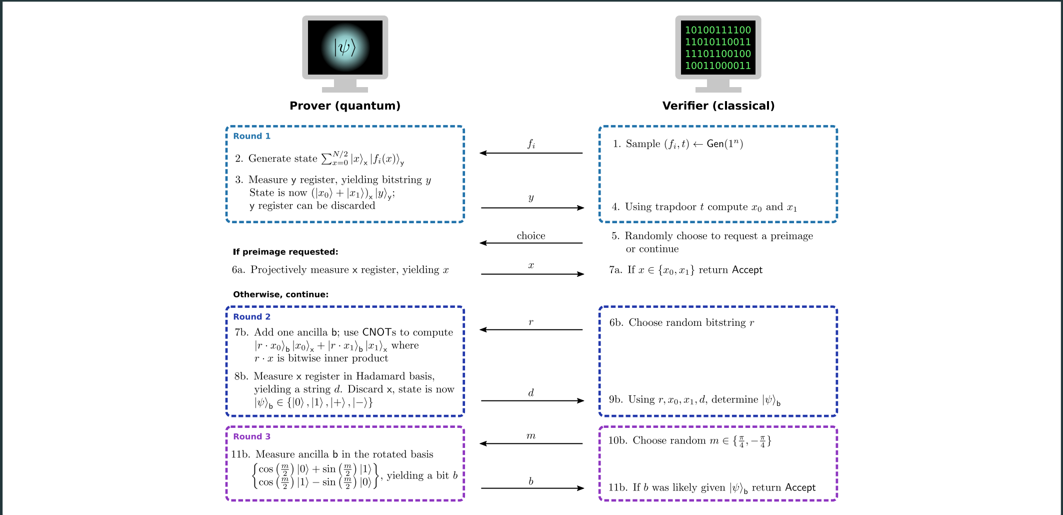
How to build a TCF?

Trapdoor [Peikert, Waters ’08; Freeman et al. ’10]: linear algebra in the exponent

Claw-free [GDKM et al. ’21 (arXiv:2104.00687)]: collisions in linear algebra in the exponent!

Full protocol



Prover (quantum)

Verifier (classical)

Round 1

1. Sample $(f_i, t) \leftarrow \mathsf{Gen}(1^n)$

2. Generate state $\sum_{x=0}^{N/2} |x\rangle_{\mathsf{x}} |f_i(x)\rangle_{\mathsf{y}}$

3. Measure y register, yielding bitstring y
State is now $(|x_0\rangle + |x_1\rangle)_{\mathsf{x}} |y\rangle_{\mathsf{y}}$;
y register can be discarded

4. Using trapdoor t compute x_0 and x_1

5. Randomly choose to request a preimage or continue

If preimage requested:

6a. Projectively measure x register, yielding x

7a. If $x \in \{x_0, x_1\}$ return Accept

Otherwise, continue:

Round 2

6b. Choose random bitstring r

7b. Add one ancilla b; use CNOTs to compute
$|r \cdot x_0\rangle_{\mathsf{b}} |x_0\rangle_{\mathsf{x}} + |r \cdot x_1\rangle_{\mathsf{b}} |x_1\rangle_{\mathsf{x}}$ where
$r \cdot x$ is bitwise inner product

8b. Measure x register in Hadamard basis, yielding a string d. Discard x, state is now $|\psi\rangle_{\mathsf{b}} \in \{|0\rangle, |1\rangle, |+\rangle, |-\rangle\}$

9b. Using r, x_0, x_1, d, determine $|\psi\rangle_{\mathsf{b}}$

Round 3

10b. Choose random $m \in \{\frac{\pi}{4}, -\frac{\pi}{4}\}$

11b. Measure ancilla b in the rotated basis
$\begin{Bmatrix} \cos\left(\frac{m}{2}\right)|0\rangle + \sin\left(\frac{m}{2}\right)|1\rangle \\ \cos\left(\frac{m}{2}\right)|1\rangle - \sin\left(\frac{m}{2}\right)|0\rangle \end{Bmatrix}$, yielding a bit b

11b. If b was likely given $|\psi\rangle_{\mathsf{b}}$ return Accept