

# How to prove you have built a quantum computer

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Gregory D. Kahanamoku-Meyer

November 2, 2023

# Introduction

... or, how did I get here?



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- Grew up and went to college in New England
- Recently completed PhD at UC Berkeley
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- Currently living on O'ahu while continuing my research remotely as a postdoc



# Quantum computing: motivation

How hard is simulating quantum systems with (regular) computers?

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Single quantum system with two states

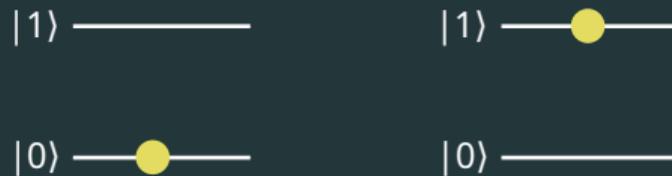
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Quantum state represented by  
2 complex numbers

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Quantum state represented by  
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Quantum state represented by  
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# Quantum computing: motivation

Complexity grows exponentially with the number of particles!

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Complexity grows exponentially with the number of particles!

Can we use that complexity to perform computations?

## Quantum computing: history

Early 90s: Theoretical algorithms for “bespoke” problems built for quantum computers

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## Simon's problem

Given a function (implemented by a **black box** or oracle)  $f : \{0, 1\}^n \rightarrow \{0, 1\}^n$  with the promise that, for some unknown  $s \in \{0, 1\}^n$ , for all  $x, y \in \{0, 1\}^n$ ,

$$f(x) = f(y) \text{ if and only if } x \oplus y \in \{0^n, s\},$$

where  $\oplus$  denotes bitwise **XOR**. The goal is to identify  $s$  by making as few queries to  $f(x)$  as possible. Note that

$$a \oplus b = 0^n \text{ if and only if } a = b$$

Furthermore, for some  $x$  and  $s$  in  $x \oplus y = s$ ,  $y$  is unique (not equal to  $x$ ) if and only if  $s \neq 0^n$ . This means that  $f$  is two-to-one when  $s \neq 0^n$ , and **one-to-one** when  $s = 0^n$ . It is also the case that  $x \oplus y = s$  implies  $y = s \oplus x$ , meaning that

$$f(x) = f(y) = f(x \oplus s)$$

Mid 90s: Theoretical algorithms for real problems!

# Quantum computing: history

Mid 90s: Theoretical algorithms for real problems!

## Grover search

Faster searching



## Shor's algorithm

Faster integer factorization

$$pq \rightarrow p \cdot q$$

## Framing the question

**Goal:** construct a physical system that can actually run these algorithms!

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Thursday, August 4, 2022  
Today's Paper

# The New York Times

World U.S. Politics N.Y. Business Opinion Tech Science Health Sports Arts Books

## Theranos Leaves Biotech Business, Turns to Building Quantum Computers

- CEO Elizabeth Holmes states the emerging field of quantum computing will be a "new start" for the company
- Despite extensive fraud at previous company, investors inexplicably believe it's a good idea to dump millions of dollars into this new venture

A photograph of Elizabeth Holmes, CEO of Theranos, speaking at a conference. She is wearing a black turtleneck and a black jacket, gesturing with her right hand. The background is a blurred purple and blue stage setting.

This is not a real headline! It is a joke.

## Framing the question

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Suppose someone opens a cloud service to perform quantum computations.

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How do we test if they are really doing anything quantum?

With only classical questions and answers?

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With only classical questions and answers?

Maybe we can use those algorithms I just mentioned?

# Grover search

Cost to find the “good” value from  $N$  indices



Quantum

$\mathcal{O}(\sqrt{N})$  operations



Classical

$\mathcal{O}(N)$  operations



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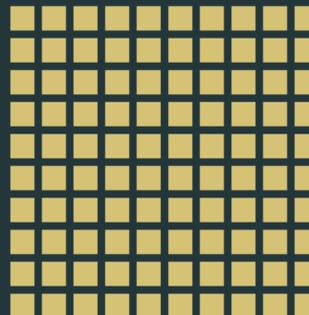
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Fewer quantum operations, but must account for differences  
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## Quantum speedups

Task	Theoretical speedup	Practical in 2023?
Grover search	Somewhat fewer ops.	Quantum computers too slow

# Shor's algorithm

Goal: factor numbers  $pq = p \cdot q$

Quantum



Classical



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# Challenge: quantum computers too noisy

Quantum information very fragile!

Quantum



Classical



## Challenge: quantum computers too noisy

Quantum information very fragile! And devices are small!

Quantum



size of largest existing device

Classical



## Quantum advantage in practice

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Is there *anything* current quantum computers can do that classical ones can't?

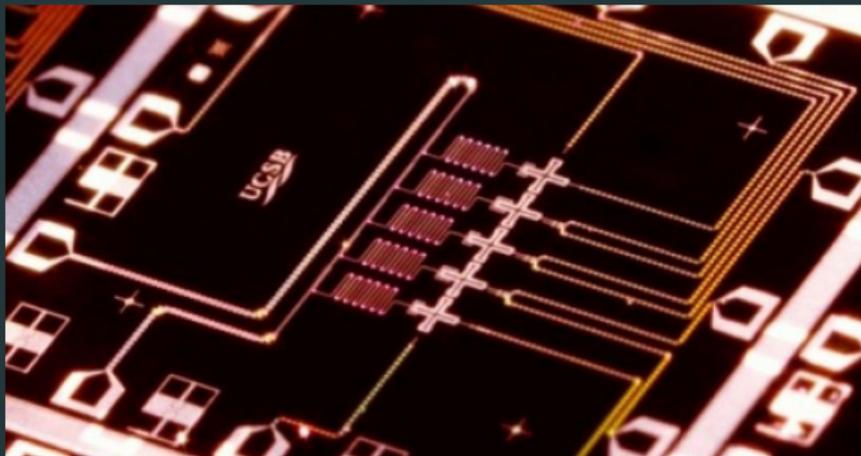
## Demonstrating “quantum advantage”

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10 years ago: nope!

Trivial to simulate!



Google/UCSB's 5-qubit chip

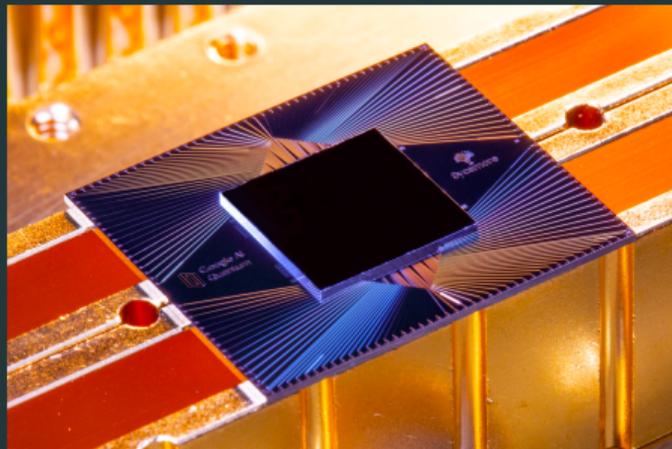
# Demonstrating “quantum advantage”

To prove we have built a quantum computer, the problem doesn't have to be *useful*

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Since 4 years ago: maybe??

Very hard to simulate!



Google's 53-qubit chip

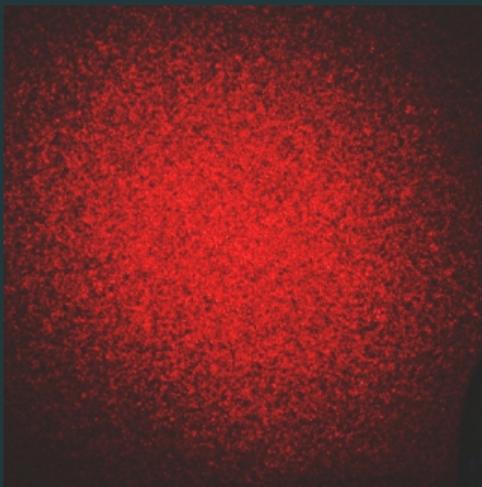
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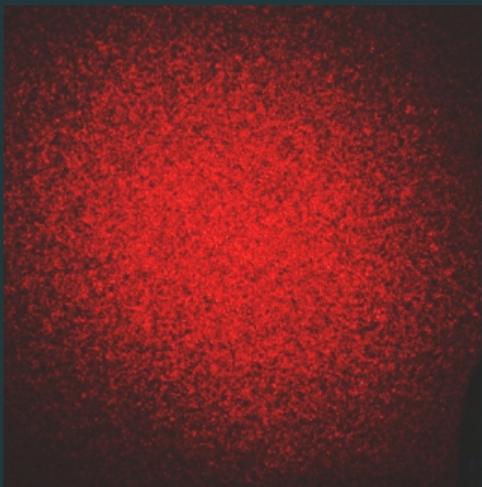
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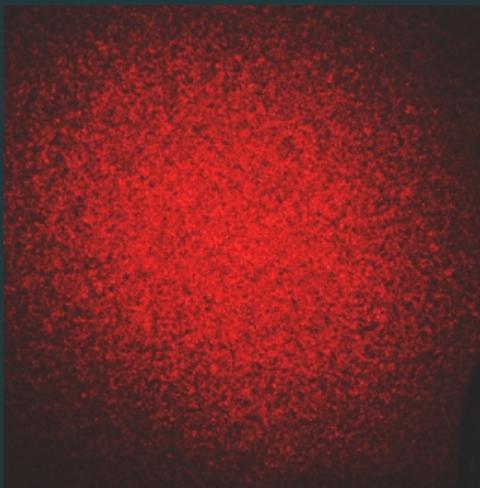
**Mathematical problem:**

1. Define some operations that generate a complicated quantum state

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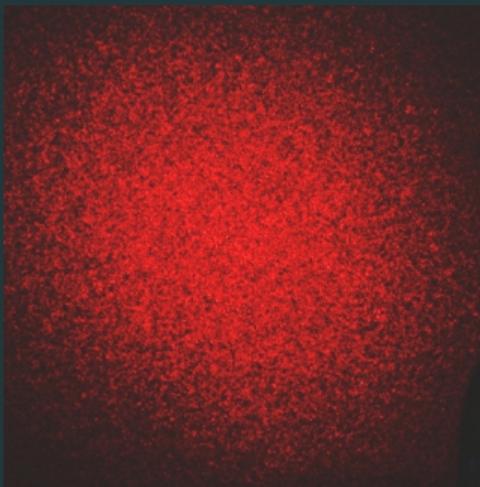
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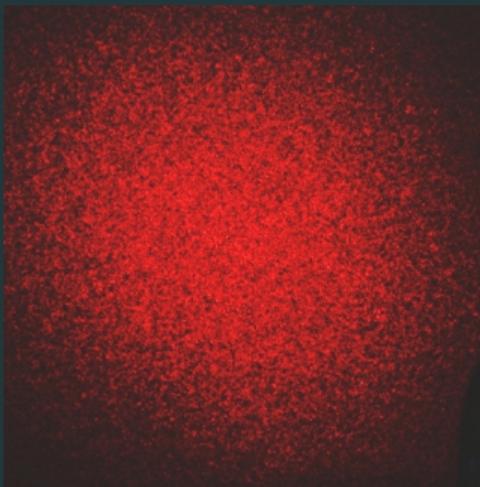
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If distribution is complicated enough, generating samples is classically hard

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## Quantum supremacy using a programmable superconducting processor

[Frank Arute](#), [Kunal Arya](#), [Ryan Babbush](#), [Dave Bacon](#), [Joseph C. Bardin](#), [Rami Barends](#), [Rupak Biswas](#), [Sergio Boixo](#), [Fernando G. S. L. Brandao](#), [David A. Buell](#), [Brian Burkett](#), [Yu Chen](#), [Ziun](#)

## A subtle challenge: verification

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How do we confirm they actually came from that distribution?

1. “Benchmark” quantum device by sampling from related but easy distributions
2. Assume nothing weird happens when you switch to the hard distribution

# Quantum advantage

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Machine learning	Depends	Too small, slow and noisy
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Random sampling	Exponentially fewer ops.	Yes, but can't check answer

# Verifiable quantum advantage

We want a problem that is hard to classically solve, but easy to classically check

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Factoring and search are such problems!

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Factoring and search are such problems!

But we also want **achievable on near-term quantum device**

# NISQ verifiable quantum advantage

NISQ = “noisy intermediate-scale quantum”

**Sampling problems**  
e.g. random circuits, Boson sampling, ...

- ✓ NISQ feasible
- ✗ Efficiently verifiable

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add structure

make less costly

**???**

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## Adding structure to sampling problems

**Example:** evolve a quantum system under “IQP” Hamiltonians (products of Pauli  $X$ 's)

$$H = X_0X_1X_3 + X_1X_2X_4X_5 + \dots \quad (1)$$

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But how sure are we that the secret is really hidden?

# The \$25 challenge

## Alice's quantum challenge

C'mon Bob, show us how quantum you really are

Alice: "Bob, do u haz qwantum?"

Bob: I haz data. Iz I qwantum?

Challenge Code

Alice's \$25 quantum challenge Posted by: mick | September 4, 2008

PAGES

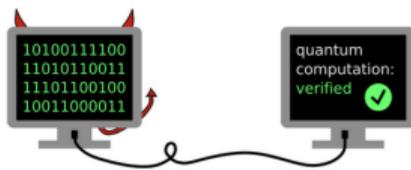
- Challenge
- Code

Hi I'm Alice (and by alice we mean mick and Dan) and this is my new blog.

My friend Bob says that he has a quantum computer and I'm not really sure I believe him, and, in a lot of ways I'm not so sure that Bob believes himself either.

It's a good thing that I came across [this paper](#) (by Dan Shephard and Michael Bremner) which

# Classical algorithm to extract secret



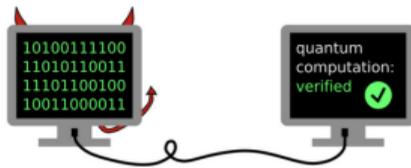
## PAPER

### Forging quantum data: classically defeating an IQP-based quantum test

Gregory D. Kahanamoku-Meyer,  
[Quantum 7, 1107 \(2023\)](#).

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## PAPER

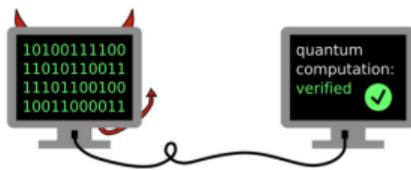
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[Bremner, Cheng, Ji 2023]: New scheme where the secret is (hopefully) hidden better

# NISQ verifiable quantum advantage

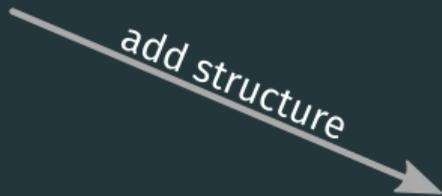
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getting the factors out as classical values is the hard part!

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Idea from cryptography: zero-knowledge proof

## Zero-knowledge proofs: differentiating colors

Challenge: Proving two balls are different colors

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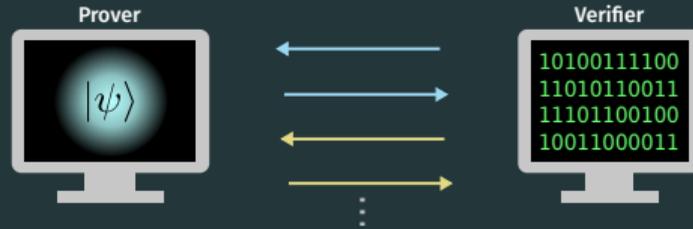
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Seeing color  $\Leftrightarrow$  Quantum capability

**Goal:** find protocol as verifiable and classically hard as factoring—  
but **less expensive** than actually finding factors (via Shor)

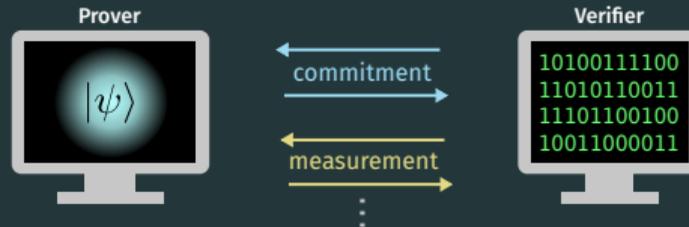
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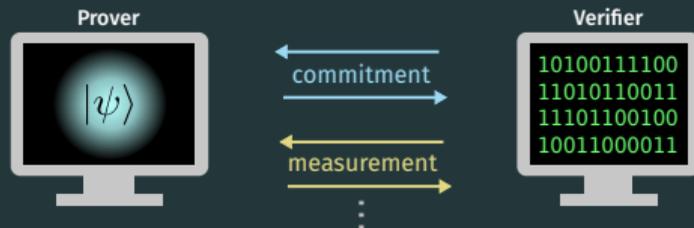


Round 1: Prover **commits** to holding a specific quantum state

Round 2: Verifier asks for **measurement** in random basis, prover performs it

# Interactive proofs of quantumness

Multiple rounds of interaction between the prover and verifier



Round 1: Prover **commits** to holding a specific quantum state

Round 2: Verifier asks for **measurement** in random basis, prover performs it

By randomizing choice of basis and repeating interaction,  
can ensure prover actually has the promised quantum state

Brakerski, Christiano, Mahadev, Vidick, Vazirani '18 (arXiv:1804.00640).

Can be extended to verify arbitrary quantum computations! (arXiv:1804.01082)

## Commitment: a secret quantum state

How does the prover commit to a state?

Consider a **2-to-1** function  $f$ :

for all  $y$  in range of  $f$ , there exist  $(x_0, x_1)$  such that  $y = f(x_0) = f(x_1)$ .

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Generate entangled superposition

$$\sum_x |x\rangle |f(x)\rangle$$



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Generate entangled superposition

$$\sum_x |x\rangle |f(x)\rangle$$

Measure 2<sup>nd</sup> register as  $y$



Pick 2-to-1 function  $f$

Store  $y$  as commitment



Prover has committed to the state  $(|x_0\rangle + |x_1\rangle) |y\rangle$

## State commitment (round 1): trapdoor claw-free functions

Prover has committed to  $(|x_0\rangle + |x_1\rangle) |y\rangle$  with  $y = f(x_0) = f(x_1)$

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Cheating classical prover can't forge the state;  
classical verifier can determine state using trapdoor.

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- **“Claw-free”**: It is cryptographically hard to find any pair of colliding inputs
- **Trapdoor**: With the secret key, easy to classically compute the two inputs mapping to any output

Cheating classical prover can't forge the state;  
classical verifier can determine state using trapdoor.

Generating a valid state without trapdoor uses  
superposition + wavefunction collapse—inherently quantum!

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**Example:** Let  $p = 5, q = 7$ ; then  $pq = 35$ .  
We have  $4^2 \equiv 11^2 \equiv 16 \pmod{35}$ ; and  $11 - 4 = 7$

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Idea: use the same circuits that we do in classical computers?

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$a$	$b$	$a \wedge b$
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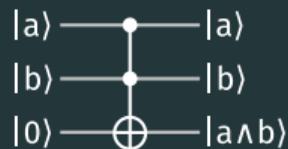
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If you're not careful, you will use up all of your precious qubits storing this "garbage data"!

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Applications include proving “quantumness” but also factoring and other algorithms!

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Using Shor's factoring algorithm to prove you are quantum:

~ 10,000,000,000 quantum operations

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Using the new protocol:

~ 2,000,000 quantum operations

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Trapped ions at the University of Maryland

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For interactive protocol, need to **measure a subset** of the quantum particles!



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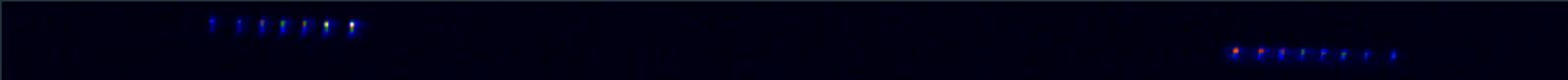
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Thank you!