A log-depth in-place quantum Fourier transform that rarely needs ancillas

[arXiv:2505.00701]

Greg Kahanamoku-Meyer [1] John Blue [1] Thiago Bergamaschi [2] Craig Gidney [3] Isaac Chuang [1]

[1]MIT [2]Berkeley [3]Google

January 28, 2026

Outline

Structure of the quantum Fourier transform

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Building a log-depth QFT with no ancillas

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Structure of the quantum Fourier transform

Classic definition of QFT on n qubits:

$$|\Phi_x\rangle \equiv \mathrm{QFT}_{2^n} |x\rangle = \sum_{z=0}^{2^n-1} e^{2\pi i x z / 2^n} |z\rangle$$

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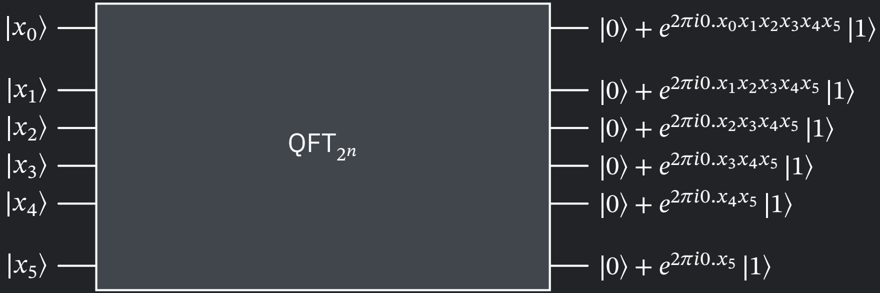
$$|\Phi_x\rangle \equiv \mathrm{QFT}_{2^n} |x\rangle = \bigotimes_{j=0}^{n-1} \left(|0\rangle + e^{2\pi i 0.x_j x_{j+1} \ldots x_{n-1}} |1\rangle\right)$$

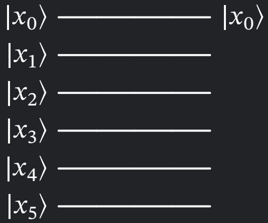
where $0.x_j x_{j+1} \cdots = 2^j x / 2^n \bmod 1$ is a binary fraction consisting of the bits of x.

Structure of the QFT



The quantum Fourier transform

Example: QFT_{2^6}



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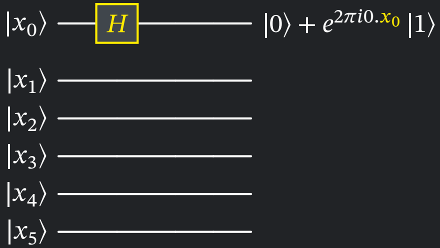

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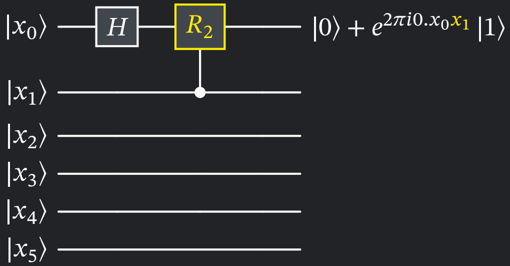

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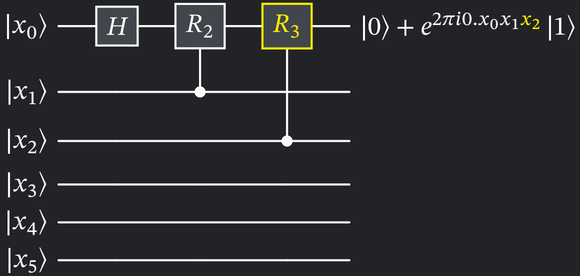

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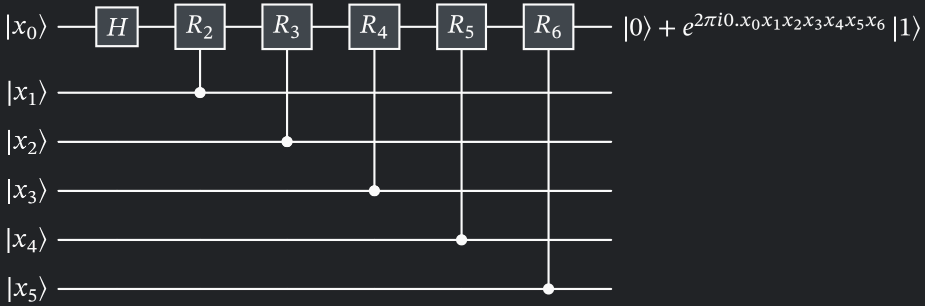

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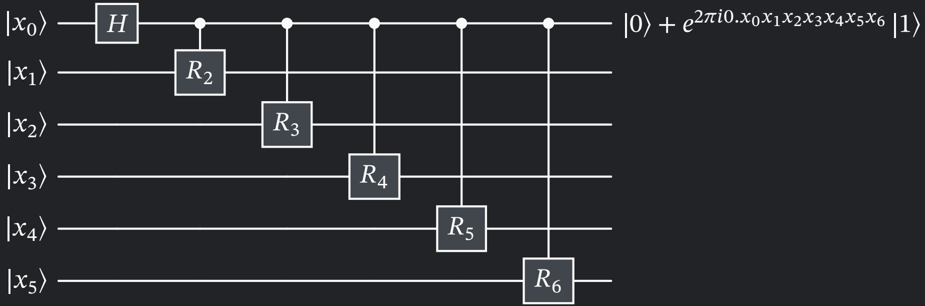

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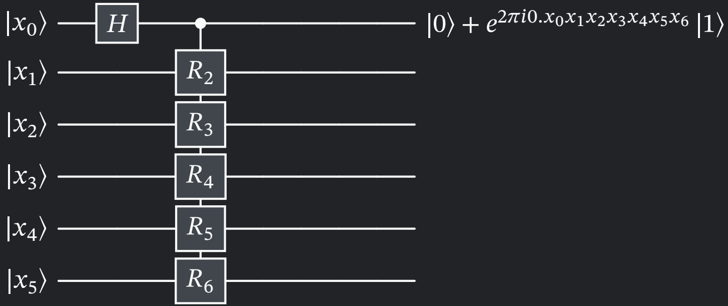

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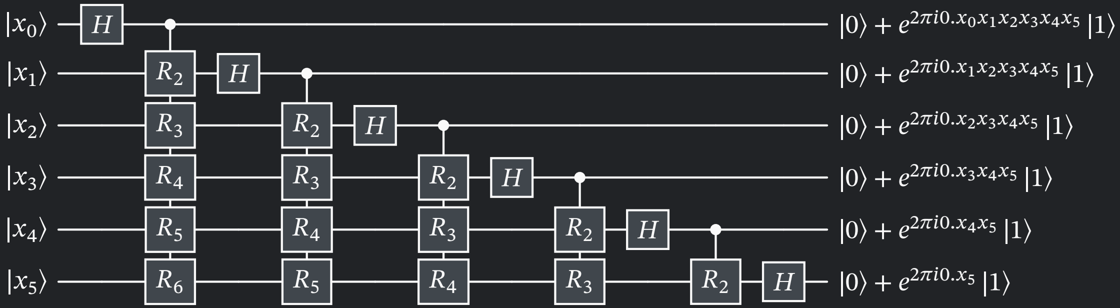
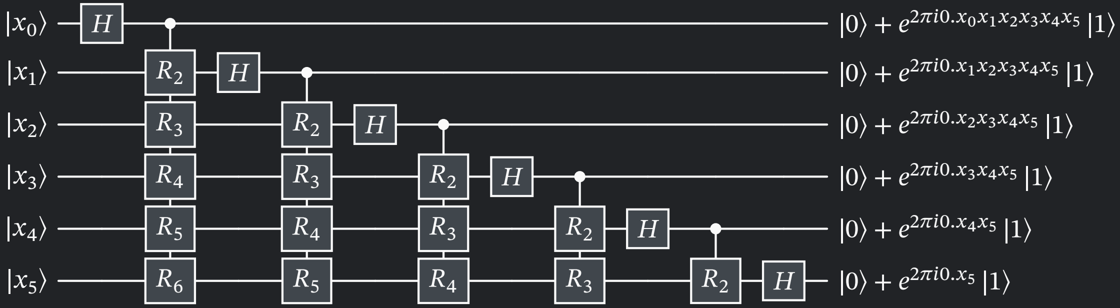


The quantum Fourier transform

Approximate QFT: truncate $0.x_j x_{j+1} \ldots$ after $m = O(\log(n/\epsilon))$ bits.



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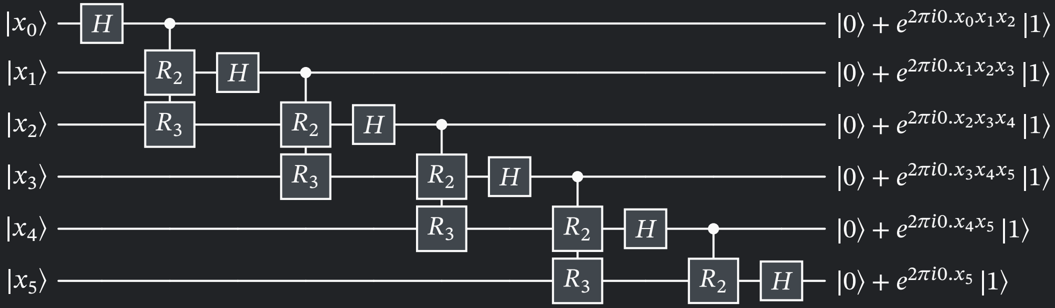

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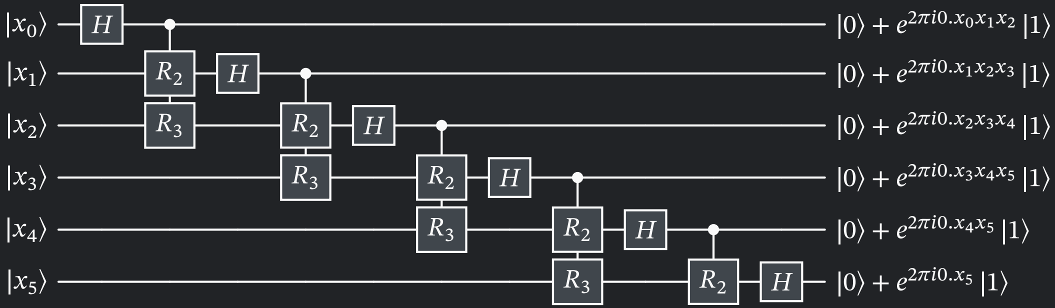
😃 **Gate count:** $O(n \log n)$



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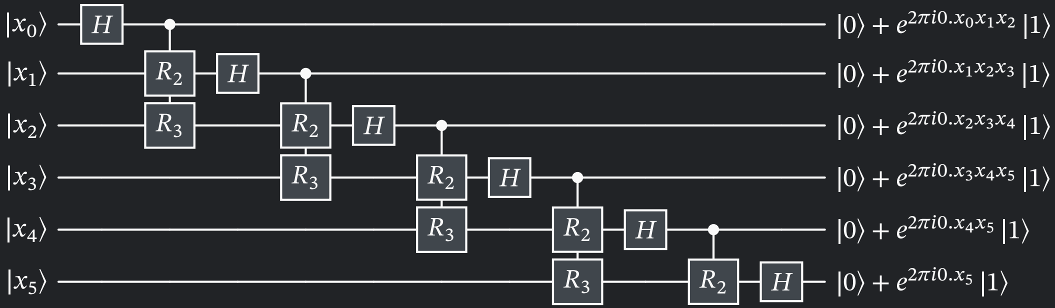
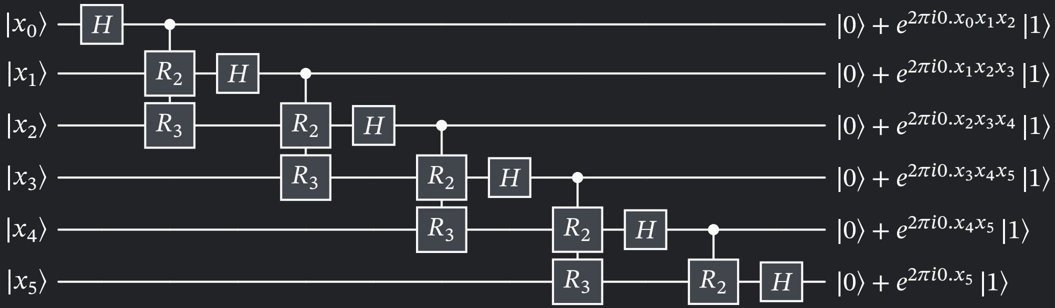


The quantum Fourier transform

😃 **Gate count:** $O(n \log n)$ 😃 **Ancillas:** 0 😢 **Circuit depth:** $O(n)$



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for any $0 < \epsilon \leq 1$

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"Surely the QFT isn't the bottleneck" -me, 2023

Some existing (approximate) QFT constructions

Coppersmith '94

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🤯 No construction with sublinear depth **and** sublinear ancilla count!

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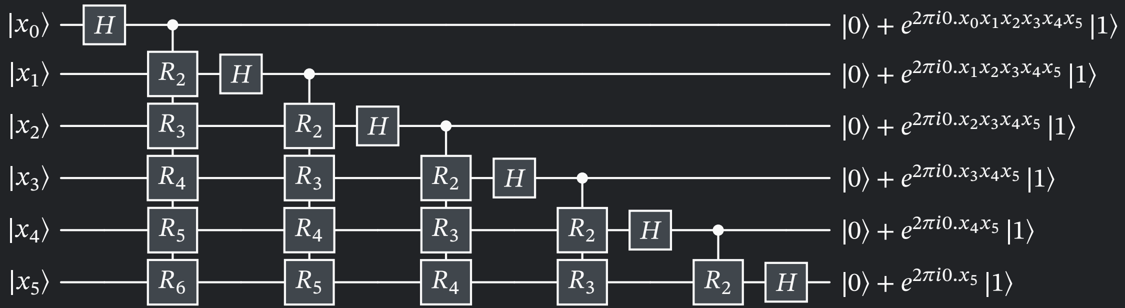

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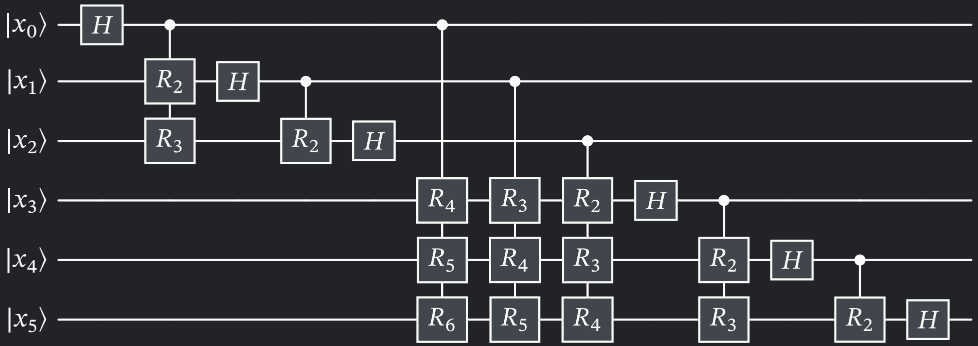

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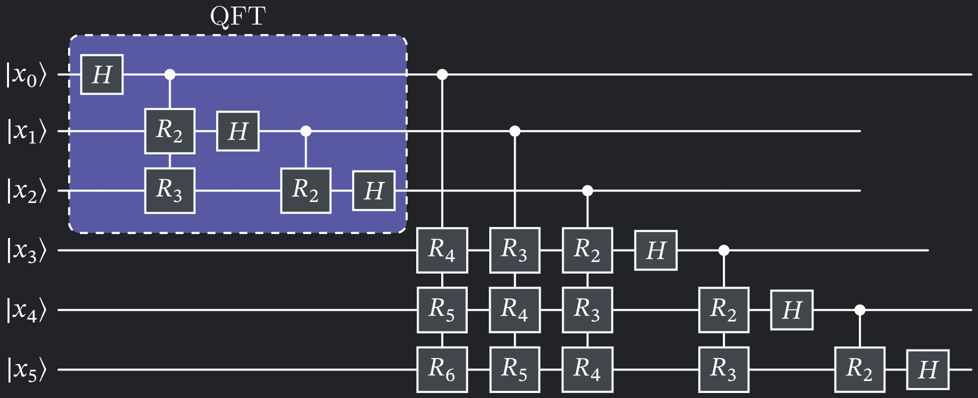

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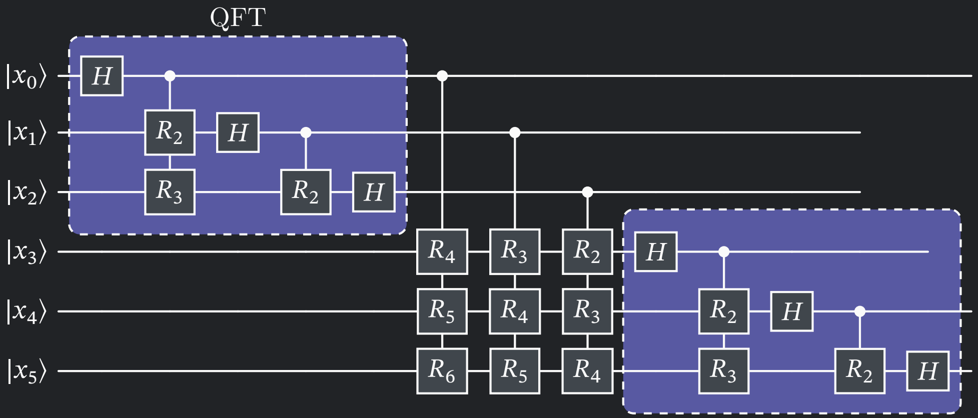

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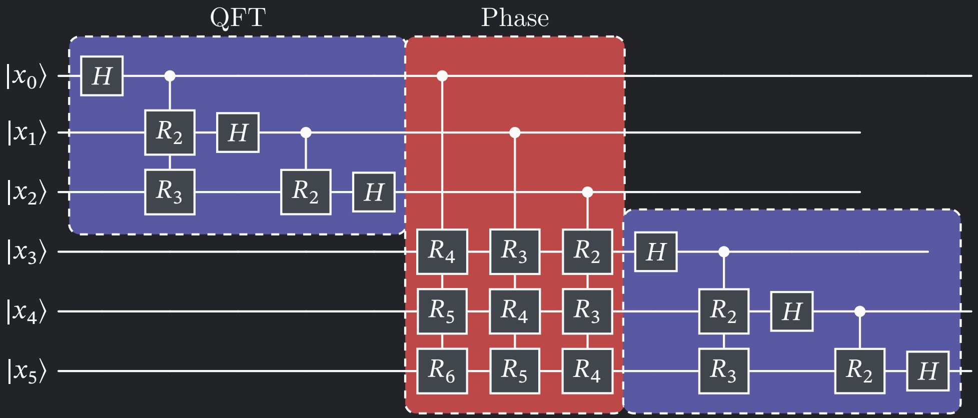

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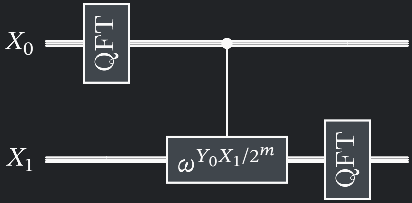
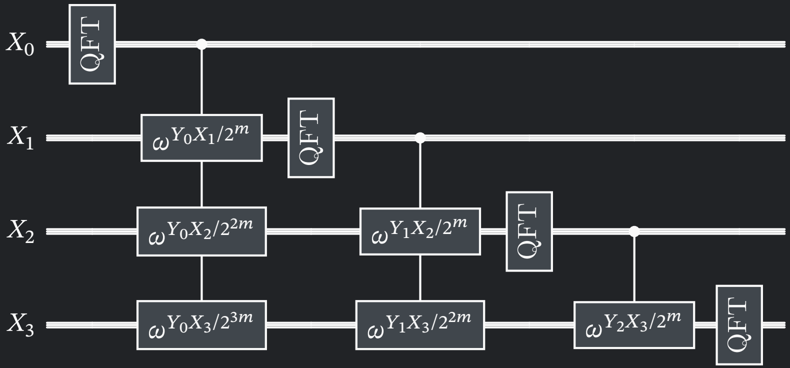


$$\omega = e^{2\pi i / 2^m}$$

The block QFT (exact!)



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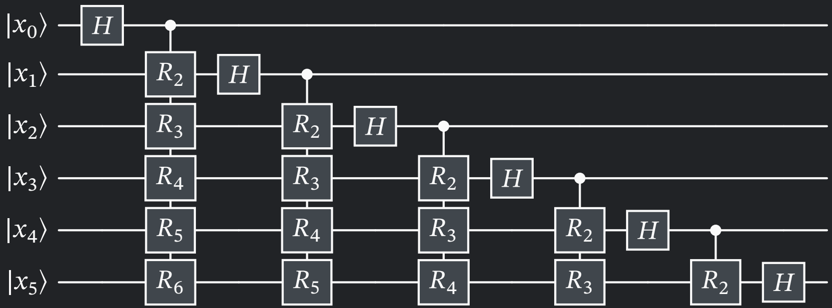
The regular QFT



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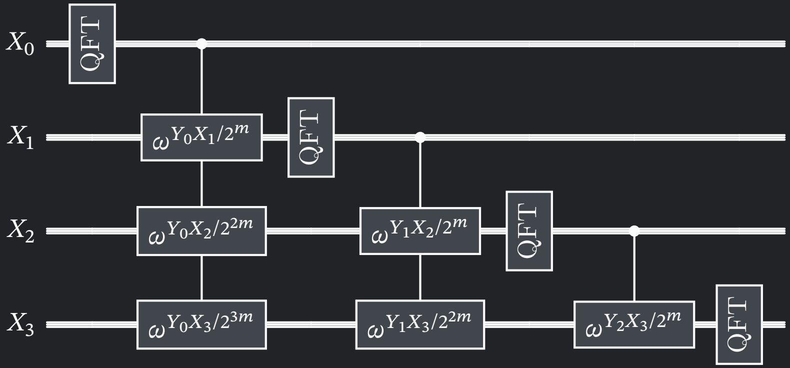

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Same structure as non-block QFT!

The quantum Fourier transform

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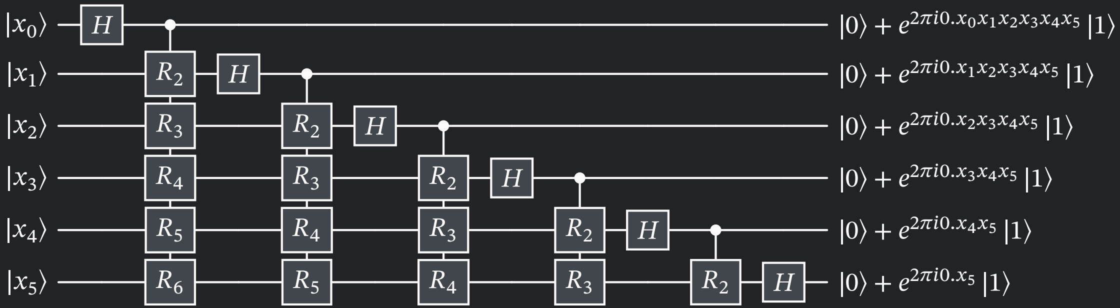

Example: approximate QFT_{2^6}

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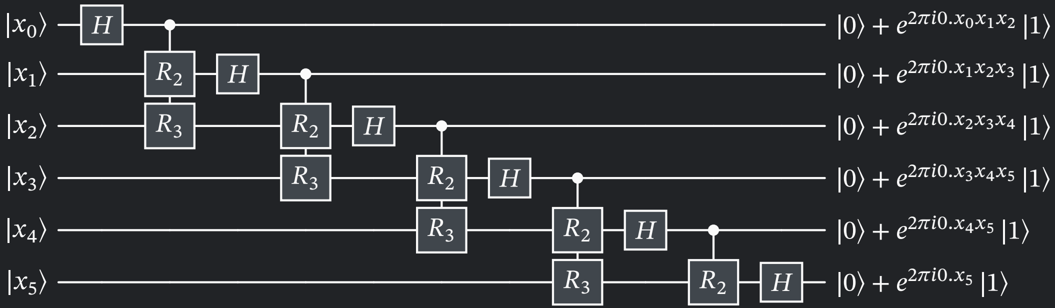

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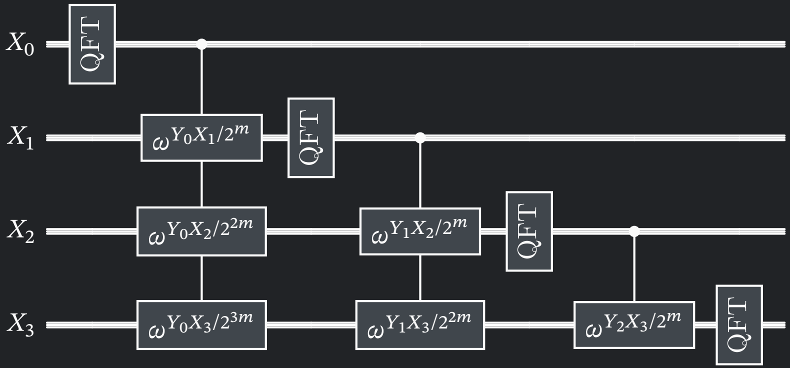

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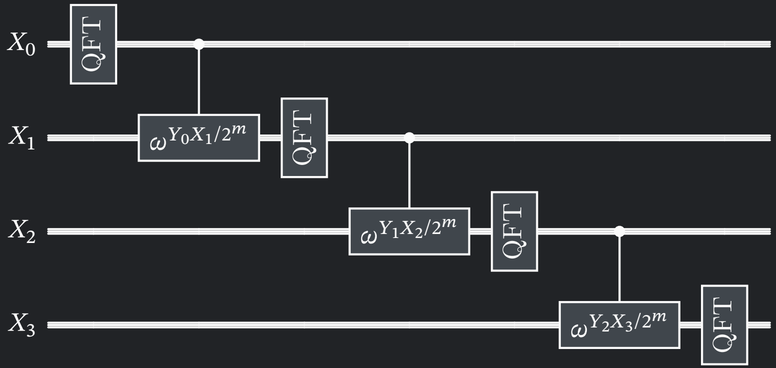

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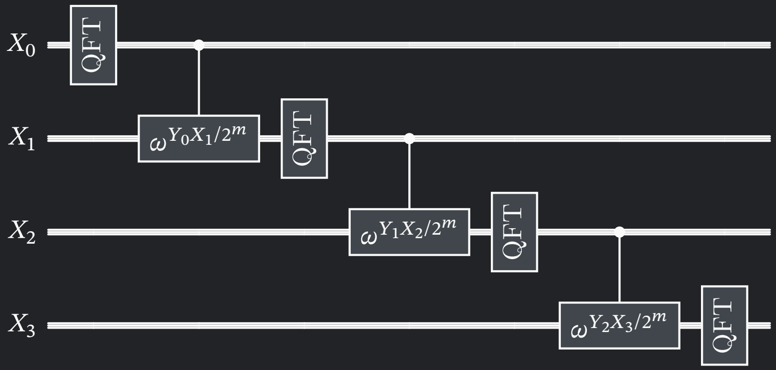
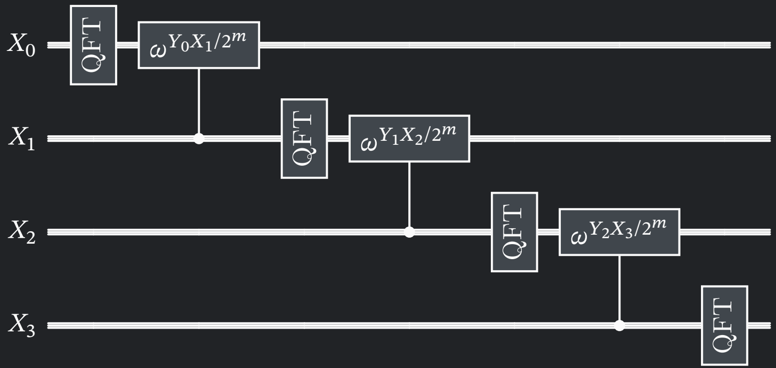


$\omega = e^{2\pi i/2^m}$

Approximation controlled by block size $m = O(\log(n/\varepsilon))$

The approximate block QFT



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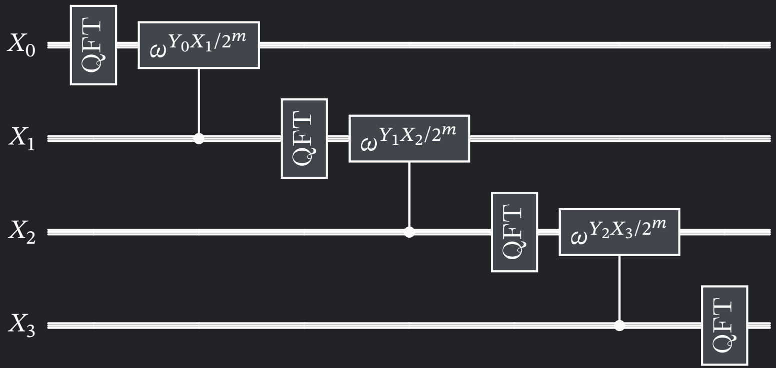

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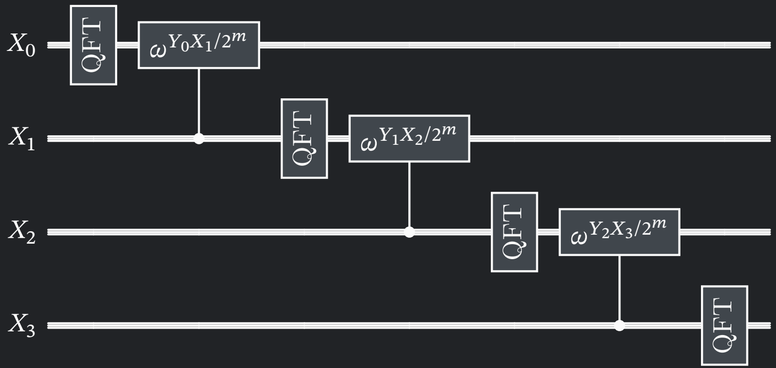
Why linear depth?

The approximate block QFT



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Why linear depth? Need $|X_i\rangle$ for block $i-1$.

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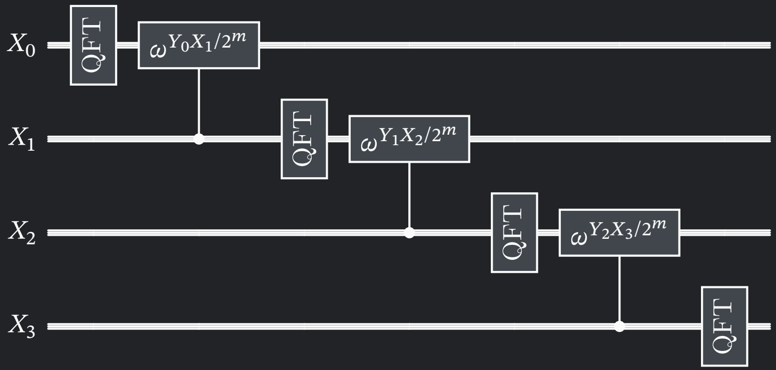

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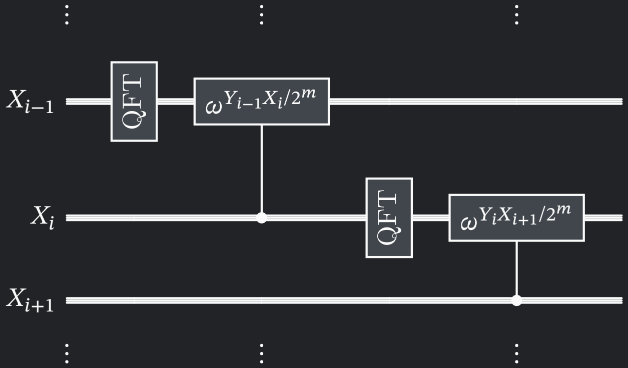
Why linear depth? Need $|X_i\rangle$ for block $i-1$. … or can we somehow recover $|X_i\rangle$?

Lowering the depth

Can we somehow recover $|X_i\rangle$?



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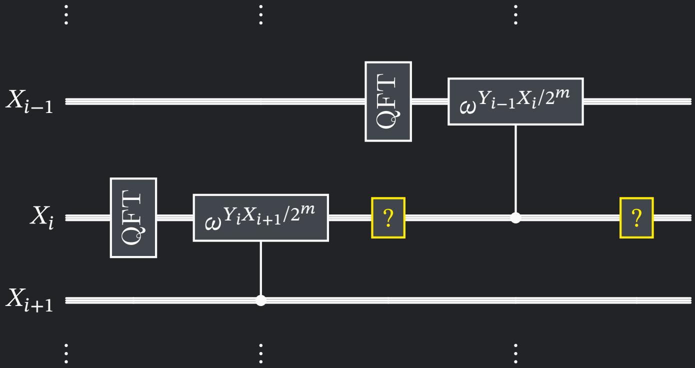

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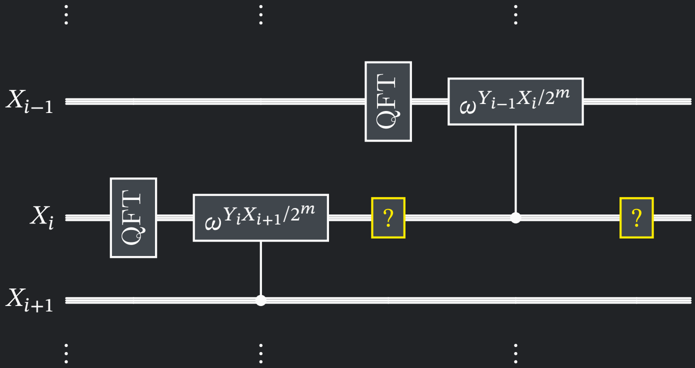

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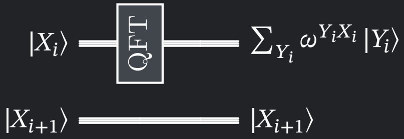

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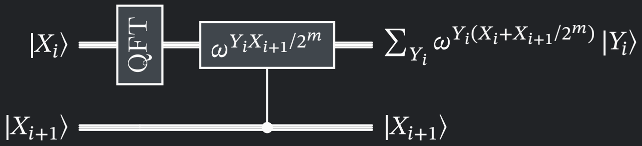

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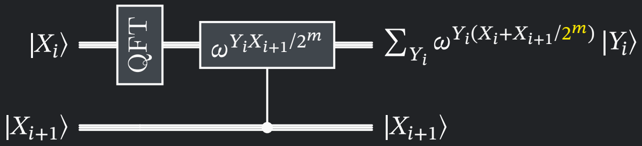

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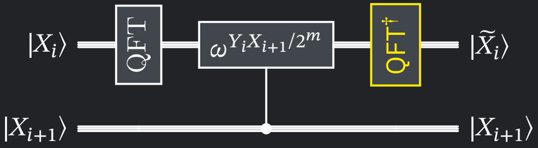

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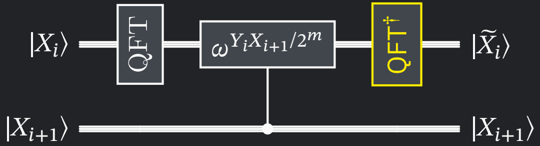
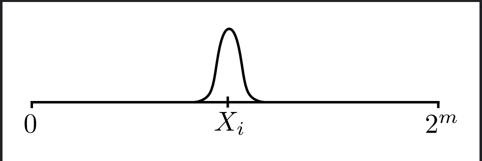
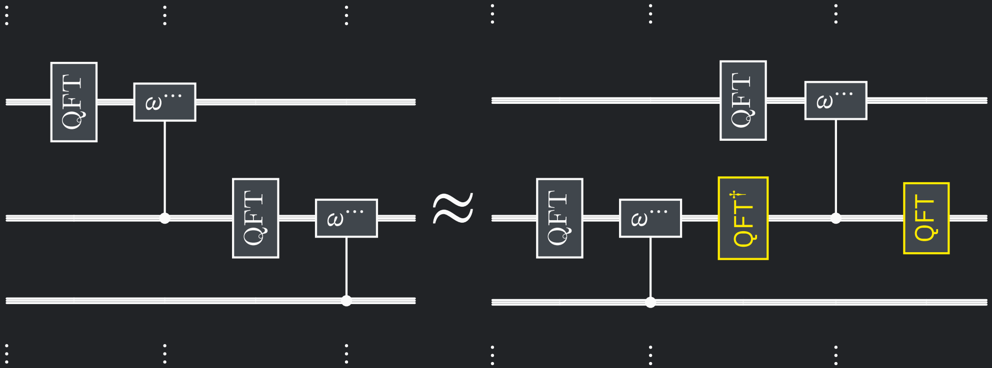


From **quantum phase estimation:**



Lowering the depth

Claim:



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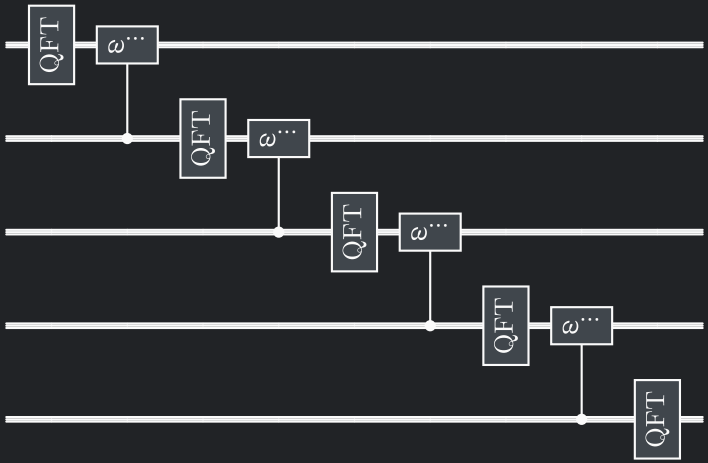

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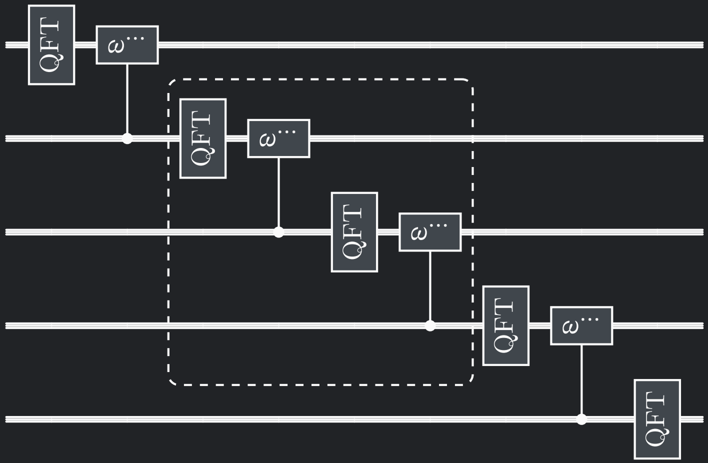

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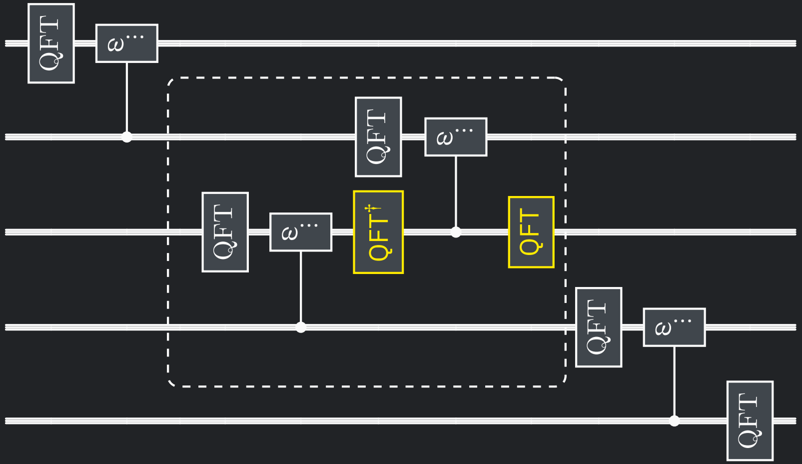

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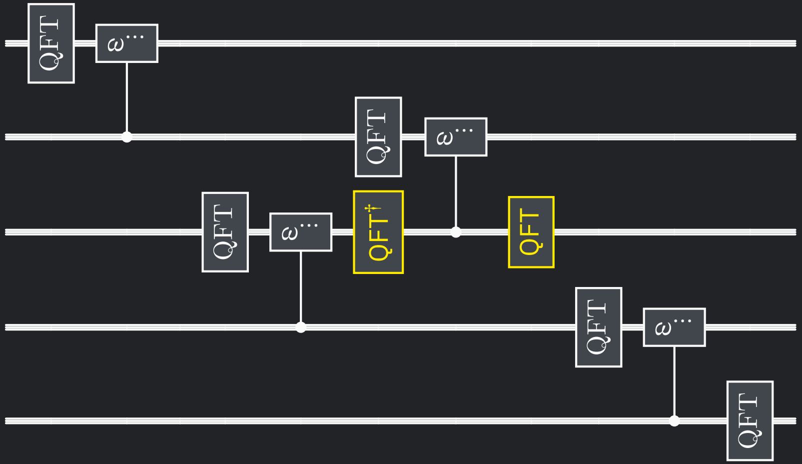

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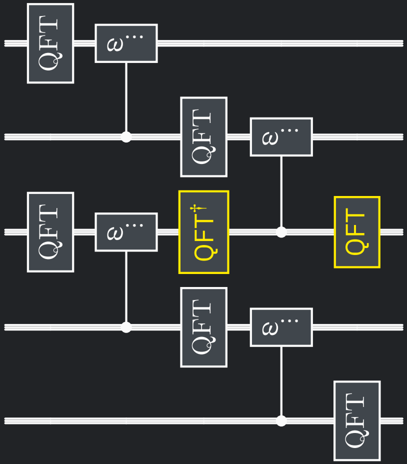
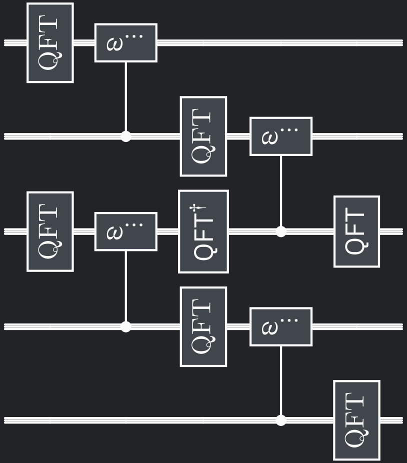


The low-depth block QFT



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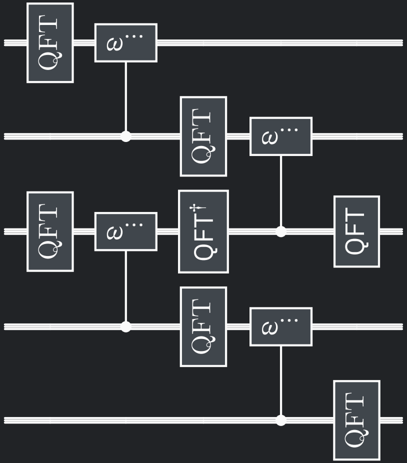

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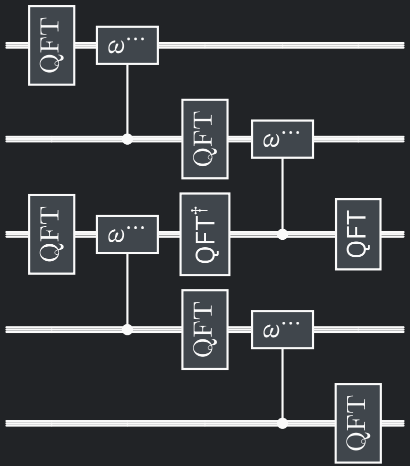

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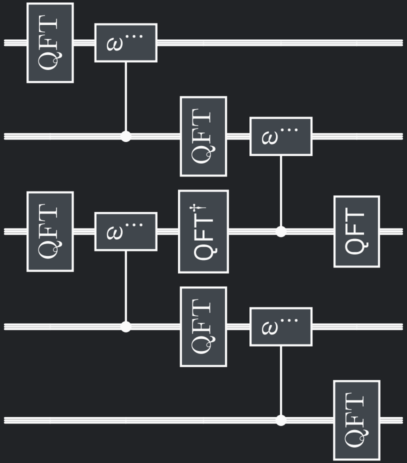

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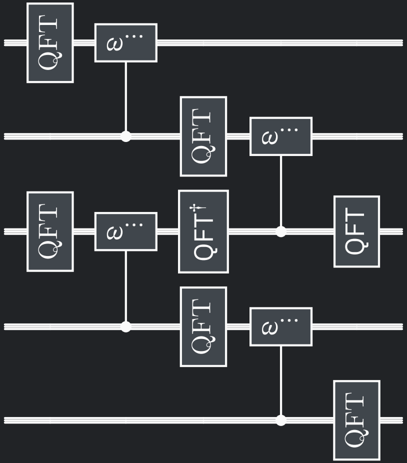

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- Depth $O(\log n/\epsilon)$
- No ancilla qubits
- Can be made *nearest-neighbor* local
- Incorrect

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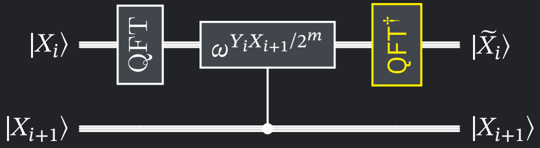
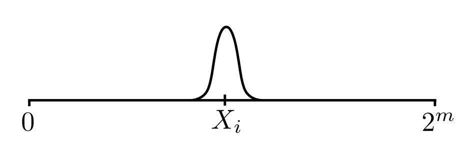


From **quantum phase estimation:**



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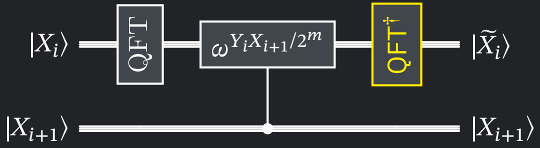

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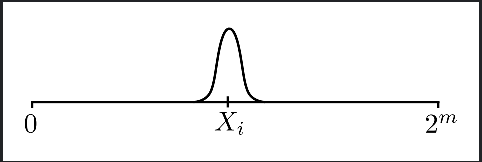

What if X_i is close to 0 or 2^m?

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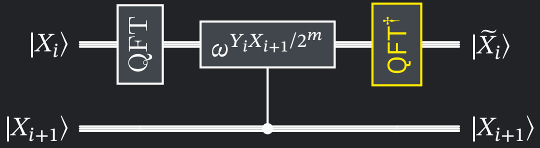

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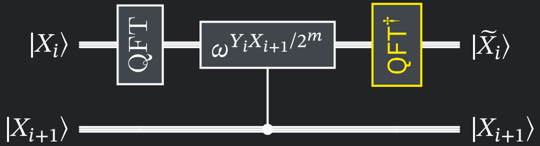

From **quantum phase estimation:**

Phase rotation that follows will be **wildly wrong!**

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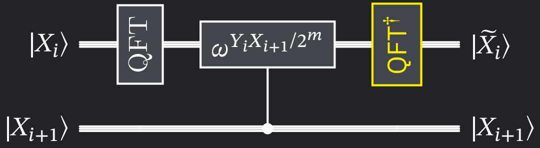

From **quantum phase estimation:**

"Wraparound" error is negligible for the vast majority of X_i

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Typical measures of error (e.g. diamond norm) consider the worst-case input

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Definition: Optimistic quantum circuits (intuitive)

C is an **optimistic circuit** with error parameter ϵ for U if $\widetilde{U}$ induced by C has

$$\|(U|\phi\rangle - \widetilde{U}|\phi\rangle\|^2 < \epsilon$$

for **most** input states $|\phi\rangle$.

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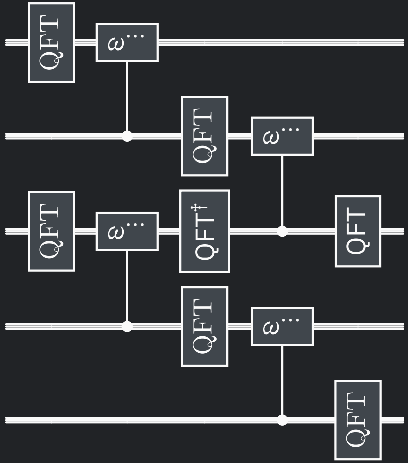
The low-depth block QFT



Theorem: this is an optimistic circuit for the quantum Fourier transform.

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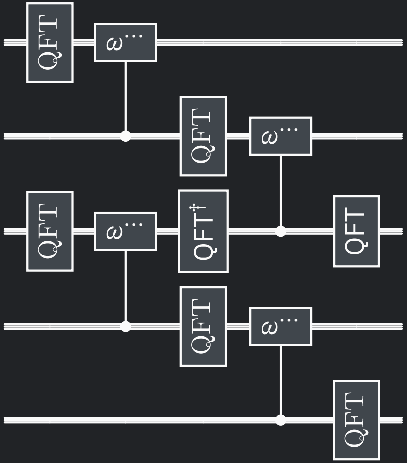

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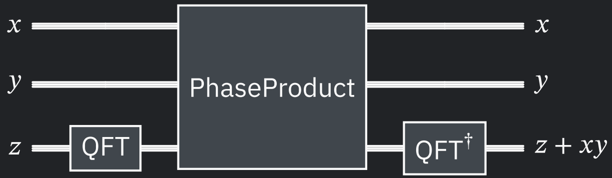
What should we do with it?



GKM, Yao '24:
PhaseProduct with...

- **Depth:** $O(n^{\epsilon})$
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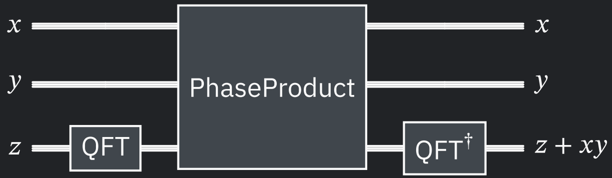
for any $0 < \epsilon \leq 1$

Using the optimistic QFT



Optimistic QFT + GKM, Yao '24:
Optimistic multiplier

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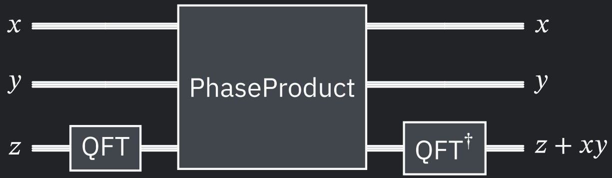

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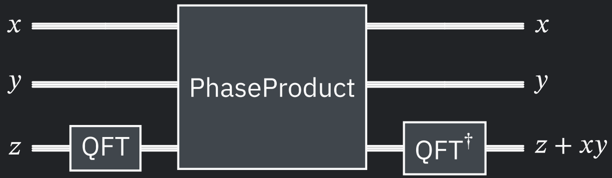

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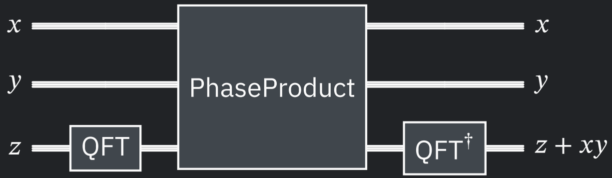

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Consequence: factoring in depth $O(n^{1+\epsilon})$ using $2n + O(n^{1-\epsilon})$ qubits

Want more factoring?

Come to the **factoring power hour** tomorrow (Thursday) at **Algorithms 6**!

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Classical

- Monte Carlo Integration
- Primality Testing
- Stochastic Gradient Descent
- And more ...

Quantum

- Gate synthesis
  - (Bocharov et al, 2014) [arXiv:1409.3552]
  - (Campbell, 2017) [arXiv:1612.02689]
- Hamiltonian Simulation
  - (Campbell, 2019) [arXiv:1811.08017]
  - (Nakaji et al, 2024) [arXiv:2302.14811]
- Quantum Signal Processing
  - (Martyn and Campbell, 2025) [arXiv:2409.03744]

Randomizing the input

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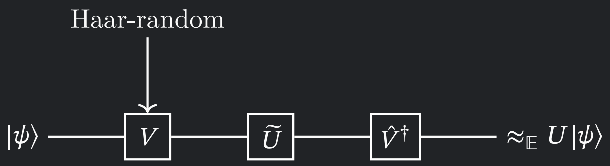
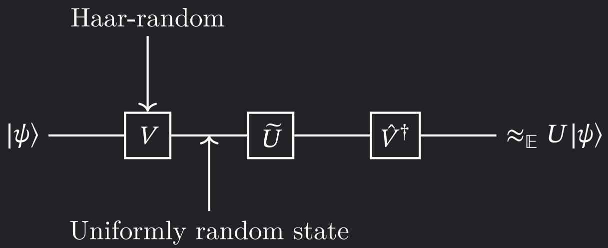
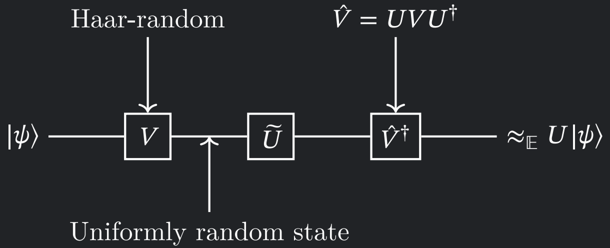



Haar-random
$|\psi\rangle$
V
$\widetilde{U}$
$\hat{V}^\dagger$
$\approx_{\mathbb{E}} U|\psi\rangle$
Uniformly random state

Randomizing the input



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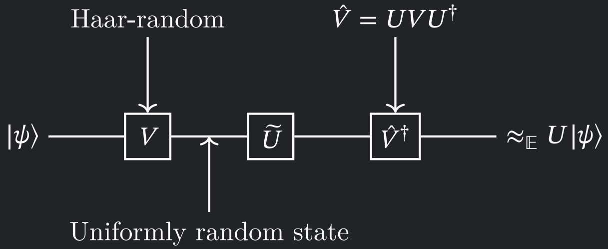

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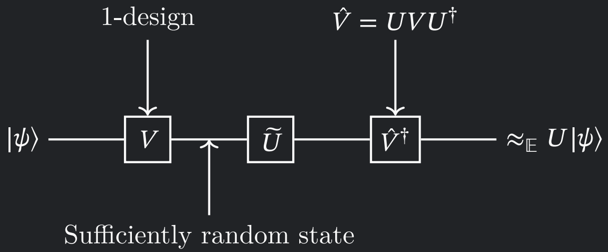

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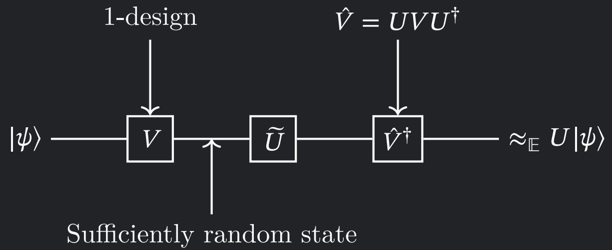

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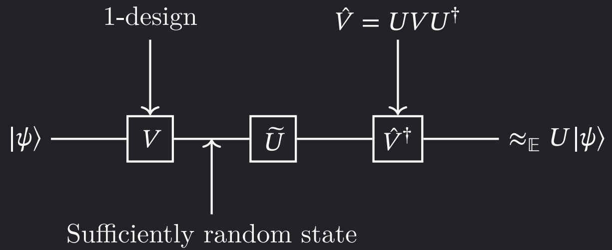

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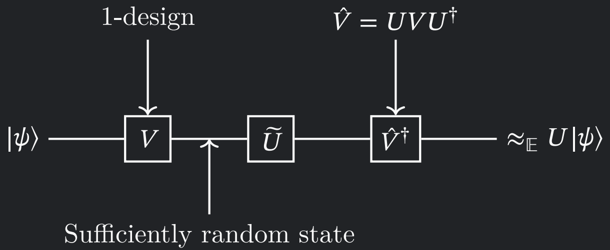

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For **QFT:** exist V and $\hat{V}^\dagger$ with depth $O(\log n)$ using $O(n/\log n)$ ancillas!

Some previous approximate QFT constructions

Coppersmith '94

😢 **Depth:** $O(n)$

😃 **Ancillas:** 0

Cleve + Watrous '00
(and follow-up works)

😃 **Depth:** $O(\log n)$

😢 **Ancillas:** $\widetilde{O}(n)$

Our results

Optimistic QFT

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*Bäumer et al. [2504.20832]: using **measurement** + **feed-forward** and $O(n)$ ancillas, can achieve nearest-neighbor connectivity

Thank you!

Greg Kahanamoku-Meyer
gkm@mit.edu

Collaborators:

John Blue

Thiago Bergamaschi

Craig Gidney

Ike Chuang